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COMBINED INFLUENCE OF HALL CURRENTS AND JOULE HEATING ON HEMODYNAMIC PERISTALTIC FLOW WITH POROUS MEDIUM THROUGH A VERTICAL TAPERED ASYMMETRIC CHANNEL WITH RADIATION

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ABSTRACT

The aim of the present attempt is hall currents and joule heating on peristaltic blood flow in porous medium through a vertical tapered asymmetric channel under the influence of radiation. The Mathematical modeling is investigated by utilizing long wavelength and low Reynolds number assumptions. The indicates an appreciable increase in the axial velocity distribution with increase in hall current parameter and porosity parameter whereas the result in axial velocity distribution diminished by increase in magnetic field parameter. The result in pressure gradient reduces by rise in hall current parameter, porosity parameter and volumetric flow rate. The temperature of the fluid increases by increase in M, N, Pr and Br and decreases by increase in m and Da.

Keywords: *Hall current, porosity parameter, joule heating, radiation and tapered channel*

1. INTRODUCTION

Past five decades researchers have enormously paid attention to the peristaltic pumping of Newtonian and non-Newtonian fluids. In particular, the discussion of peristaltic fluid flow has created lots of curiosity and hence, good literature is currently available on the subject. A thorough understanding of peristalsis is of great interest, due to its natural property of the numerous biological systems taking sleek muscle tubes which transport biofluids through its propulsive movements. It is found within the movement of food bolus through oesophagus, transport of body waste from urinary organ to the bladder through the channel, the movement of spermatozoa within the ducts afferents of the male reproductive tract, circulation of blood in the small blood vessels, the movement of chyme in the gastrointestinal tract, intra-uterine fluid motion, movements of ovum in the female fallopian tube are some samples of peristaltic fluid flow. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems. Several attempts had been made to know and understand peristaltic action in different situations. Since the first attempt of Latham (1966) some interesting investigations in this direction have been given in (Fung and Yih, 1968; Shapiro *et al*., 1969; Jaffrin and Shapiro, 1971; Srivastava and Srivastava, 1984; Agarwal and Anwaruddin, 1984; Shehawey and Husseny, 2002; Siddiqui *et al.,* 2004; El Naby *et al*., 2006; Mekheimer, 2004; Mishra and Ramachandra Rao, 2003; Haroun, 2007; Vajravelu *et al*., 2011).

The Hall effect is important when the Hall parameter which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency is high; this can occur if the collision frequency is low or when the magnetic field is high. This is a current trend in MHD because of its important influence of the electromagnetic force. Hence, it is important to study Hall effects and heat transfer effects on the flow to be able to determine the efficiency of some devices such as power generators and heat exchangers. Attia (2004) had examined unsteady Hartmann flow with heat transfer of a viscoelastic fluid taking the Hall effect into account. Asghar *et al.* (2005) studied the effects of Hall current and heat transfer on flow due to a pull of eccentric rotating disk. Hayat *et al*. (2007) studied the Hall effects on the peristaltic flow of a Maxwell fluid in a porous medium. Hayat *et al*. (2016) investigated an influence of Hall current and chemical reaction in mixed convective peristaltic flow of Prandtl fluid. For the benefit of the readers in this direction, you can take these additional papers (Akbar and Nadeem, 2010, Ali *et al*., 2010; Eldabe *et al*., 2015, Nowar, 2014, Abd El-Maboud and Mekheimer, 2010, Mekheimer *et al*., 2010; Gharsseldien *et al*., 2010; Srinivas and Muthuraj, 2011) into account.

The study of the flow of fluids through porous media has the greatest significance for the last one and half centuries because of their occurrence in Sanitary Engineering and newly emerging fields such as Petroleum Industry, Polymer Industry and its wide applications in Physiology and Bio-Mechanics. The petroleum industries are showing good curiosity in the study of the heat and mass transfer of the flow of oil through porous rocks. In order to study the seepage of water in river beds, one requires the knowledge of the flow of fluids through porous media, which is necessary to tap groundwater from the underground reservoirs to overcome the crisis created by the failure of monsoons in the rain-fed areas. Flow through porous media is also useful to the paper industry because, in the papermaking process, felts are used to carry the paper sheet through its many stages of formation and drying.

In general heat transfer, plays a vital role in MHD flows. Heat transfer is that the transition of thermal energy from a section of upper temperature to a locality of lower temperature. Vajravelu *et al*. (2007) gave a numerical model for peristaltic transportation and heat transfer in a perpendicular porous annulus. In another try, Nadeem *et al*. (2009) mentioned on an impact of heat transfer on peristalsis with

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variable viscosity. Shehzad *et al*. (2014) investigated on hydromagnetic peristaltic transportation of nanofluid with Joule Heating and thermophoresis. The peristaltic flow of fourth-grade fluid with Dufour and Sore effects was studied by Mustafa *et al*. (2014). Recently, Ravi Kumar (2016) investigated on hydromagnetic peristaltic transportation through a perpendicular tapered uneven channel with radiation. Some pertinent studies on this topic will be found from the list of Refs. Such as Tasawar Hayat *et al*. (2017), Kothandapani *et al*. (2015) Abzal *et al*. (2016) , Jitendra Kumar Singh *et al*. (2016) and Veeresh *et al*. (2017).

2. FORMULATION OF THE PROBLEM

We consider the MHD peristaltic transport of an incompressible viscous fluid in a two-dimensional uneven inclined perpendicular tapered channel under the influence of porous medium. The joule heating, hall currents and radiation is taken into the account. The left wall of the channel is maintained at temperature T_0, whereas the right wall has temperature T_1. We tend to assume that the fluid is subject to a relentless transverse magnetic field B_0. The fluid is induced by sinusoidal wave trains propagating with constant speed *c* along the channel walls.

The geometry of the wall deformations are drawn by the subsequent expressions

$$Y = \overline{H_2} = b + m^I \bar{X} + d \sin\left[\frac{2\pi}{\lambda}(\bar{X} - c\,\bar{t})\right] \tag{1}$$

$$Y = \overline{H_1} = -b - m^I \bar{X} - d \sin\left[\frac{2\pi}{\lambda}(\bar{X} - c\,\bar{t}) + \phi\right] \tag{2}$$

Where b is the half-width of the channel, d is the wave amplitude, c is the phase speed of the wave and m' $(m' << 1)$ is the non-uniform parameter, λ is the wavelength, t is the time and X are the direction of wave propagation. The phase difference ϕ varies in the range $0 \le \phi \le \pi$, $\phi = 0$ corresponds to the symmetric channel with waves out of phase and further b, d and ϕ satisfy the following conditions for the divergent channel at the inlet $d \cos\left(\frac{\phi}{2}\right) \le b$. It is assumed that the left wall of the channel is maintained at temperature T_0 while the right wall has temperature T_1.

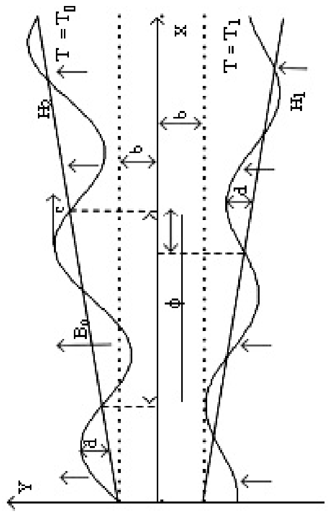

Fig. 1 Schematic diagram of the physical model

The equations governing the motion for the present problem prescribed by

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

The momentum equations are

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + u\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \left[\frac{\sigma B_0^2}{1+m^2}\right]\left(mv - (u+c)\right) - \frac{\mu}{k_1}(u+c) \tag{4}$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + u\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \left[\frac{\sigma B_0^2}{1+m^2}\right]\left(m(u+c) + v\right) - \frac{\mu}{k_1}(v+c) \tag{5}$$

The energy equation is

$$\rho\, C_p\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]T = k\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]T + Q_0 + \sigma B_0^2 u^2 - \frac{\partial q}{\partial x} \tag{6}$$

u and v are the velocity components in the corresponding coordinates, k_1 is the permeability of the porous medium,ρ is the density of the fluid, p is the fluid pressure, *k* is the thermal conductivity, μ is the coefficient of the viscosity, Q_0 is the constant heat addition/absorption, C_p is the specific heat at constant pressure, σ is the electrical conductivity, *g* is the acceleration due to gravity and T is the temperature of the fluid.

The relative boundary conditions are

$\bar{U} = 0, \bar{T} = T_0$ at $\bar{Y} = \bar{H}_1$

$\bar{U} = 0, \bar{T} = T_1$ at $\bar{Y} = \bar{H}_2$

The radioactive heat flux (Cogley et al (1968)) is given by

$$\frac{\partial q}{\partial x} = 4\alpha^2(T_0 - T_1) \tag{7}$$

here α is the mean radiation absorption coefficient.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$x = X - ct$, $y = Y$, $u = U - c$, $v = V$ and $p(x) = P(X, t)$ (8)

Introducing the following non-dimensional quantities:

$$\bar{x} = \frac{x}{\lambda} \quad \bar{y} = \frac{y}{b} \quad \bar{t} = \frac{ct}{\lambda} \quad \bar{u} = \frac{u}{c} \quad \bar{v} = \frac{v}{c\delta} \quad h_1 = \frac{H_1}{b} \quad h_2 = \frac{H_2}{b}$$

$$p = \frac{b^2 p}{c\lambda\mu} \quad \theta = \frac{T - T_0}{T_1 - T_0} \quad \delta = \frac{b}{\lambda} \quad \mathrm{Re} = \frac{\rho c b}{\mu} \quad \eta_1 = \frac{\rho a_0^3 g}{\lambda \mu c}$$

$$M = B_0 b\sqrt{\frac{\sigma}{\mu}} \quad \Pr = \frac{\mu C_p}{k} \quad E_c = \frac{c^2}{C_p(T_1 - T_0)} \quad N^2 = \frac{4\alpha^2 d_1^2}{k}$$

$$\gamma = \frac{Q_0 b^2}{\mu C_p (T_1 - T_0)} \quad \varepsilon = \frac{d}{b} \quad \eta = \frac{\rho a_0^2 g}{\mu c} \tag{9}$$

where $\epsilon = \frac{d}{b}$ is the non-dimensional amplitude of channel, $\delta = \frac{b}{\lambda}$ is the wave number,$k_1 = \frac{\lambda m^I}{b}$ is the non - uniform parameter, Re is the Reynolds number, M is the Hartman number, $K = \frac{k}{b^2}$ Permeability parameter, Pr is the Prandtl number, E_c is the Eckert number, γ is the heat source/sink parameter, B_r (= E_cP_r) is the Brinkman number, η andη_1 are gravitational parameters and N^2 is the radiation parameter.

3. SOLUTION OF THE PROBLEM

In view of the above transformations (8) and non-dimensional variables (9), equations (3-6) are reduced to the following non-dimensional form after dropping the bars,

$$Re\delta\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{M^2}{1+m^2}\left(m\delta v - (u+1)\right) - \frac{1}{D}u - \frac{1}{D} \tag{10}$$

$$Re\delta^3\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\delta M^2}{1+m^2}\left(m(u+1) + \delta v\right) - \delta^2\frac{1}{D}v - \delta^2\frac{1}{D} \tag{11}$$

$$Re\left[\delta u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right] = \frac{1}{P_r}\left[\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right] + \beta + M^2E_cu^2 + \frac{N^2\theta}{P_r} \tag{12}$$

Applying long wave length approximation and neglecting the wave number along with low-Reynolds numbers. Equations (10-12) become

$$\frac{\partial^2 u}{\partial y^2} - \left(\frac{M^2}{1+m^2} + \frac{1}{D}\right) u = \frac{\partial p}{\partial x} + \left(\frac{M^2}{1+m^2} + \frac{1}{D}\right) \quad (13)$$

$$\frac{\partial p}{\partial y} = 0 \quad (14)$$

$$\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + \beta + M^2 E_c u^2 + \frac{N^2\theta}{P_r} = 0 \quad (15)$$

The relative boundary conditions in dimensionless form are given by

u = -1, θ = 0 at $y = h_1 = 1 - k_1 x - \varepsilon \sin\left[2\pi(x-t) + \phi\right]$ (16)

u = -1, θ = 1 at $y = h_2 = 1 + k_1 x + \varepsilon \sin\left[2\pi(x-t)\right]$ (17)

The solutions of velocity and temperature with subject to boundary conditions (16) and (17) are given by

$$u = a_1 \sinh[\alpha_1 y] + \cosh[\alpha_1 y] + a \quad (18)$$

Where

$$a_1 = \left(\frac{(1+a)}{\sinh[\alpha_1 h_2]}\right) + \left[\frac{\cosh[\alpha_1 h_2]\,(1+a)}{(\sinh[\alpha_1 h_2])\left(\frac{\cosh[\alpha_1 h_1] - \cosh[\alpha_1 h_2]}{\sinh[\alpha_1 h_1] - \sinh[\alpha_1 h_2]}\right)\sinh[\alpha_1 h_1] - \cosh[\alpha_1 h_1]}\right]$$

$$a_2 = \left[\frac{(1+a)}{\left(\frac{\cosh[\alpha_1 h_1] - \cosh[\alpha_1 h_2]}{\sinh[\alpha_1 h_1] - \sinh[\alpha_1 h_2]}\right)\sinh[\alpha_1 h_1] - \cosh[\alpha_1 h_1]}\right]$$

$a = -\left[1 + \left(\frac{p}{\frac{M^2}{1+m^2} + \frac{1}{Da}}\right)\right]$, where $p = \frac{\partial p}{\partial x}$

$$\theta = a_9 \cos[N\,y] + a_8 \sin[Ny] - \frac{\beta P_r}{N^2} - \left[\frac{M^2 B_r a_3}{4\alpha_1^2 + N^2}\right] e^{2\alpha_1 y} - \left[\frac{M^2 B_r a_4}{4\alpha_1^2 + N^2}\right] e^{-2\alpha_1 y} - \left[\frac{M^2 B_r a_5}{\alpha_1^2 + N^2}\right] e^{\alpha_1 y} - \left[\frac{M^2 B_r a_6}{\alpha_1^2 + N^2}\right] e^{-\alpha_1 y} - \frac{M^2 B_r a_7}{N^2} \quad (19)$$

Where

$$a_3 = \left[\frac{a_1^2}{4} + \frac{a_2^2}{4} + \frac{a_1 a_2}{2}\right], a_4 = \left[\frac{a_1^2}{4} + \frac{a_2^2}{4} - \frac{a_1 a_2}{2}\right]$$

$$a_5 = [a(a_1 + a_2)], a_6 = [a(a_2 - a_1)]$$

$$a_7 = \left[-\frac{a_1^2}{2} + \frac{a_2^2}{2} + a^2\right]$$

$$a_8 = \left[\frac{-\cos[Nh_1] - a_{10} - a_{11} - a_{12} - a_{13} - a_{14} - a_{15}}{a_{16}}\right]$$

$$a_9 = \left[\frac{-a_8\ \sin[Nh_1] + \frac{\beta P_r}{N^2} + \left[\frac{M^2 B_r a_3}{4\alpha_1^2 + N^2}\right] e^{2\alpha_1 h_1}}{\cos[Nh_1]}\right] + \left[\frac{\left[\frac{M^2 B_r a_4}{4\alpha_1^2 + N^2}\right] e^{-2\alpha_1 h_1} + \left[\frac{M^2 B_r a_5}{\alpha_1^2 + N^2}\right] e^{\alpha_1 h_1} + \left[\frac{M^2 B_r a_6}{\alpha_1^2 + N^2}\right] e^{-\alpha_1 h_1} + \frac{M^2 B_r a_7}{N^2}}{\cos[Nh_1]}\right]$$

$$a_{10} = \left[\left(\frac{\beta P_r}{N^2}\right)(\cosh[Nh_1] - \cosh[Nh_2])\right]$$

$$a_{11} = \left[\left(\frac{M^2 B_r a_3}{4\alpha_1^2 + N^2}\right)(e^{2\alpha_1 h_2}\cosh[Nh_1]\ - e^{2\alpha_1 h_2}\cosh[Nh_1])\right]$$

$$a_{12} = \left[\left(\frac{M^2 B_r a_4}{4\alpha_1^2 + N^2}\right)(e^{-2\alpha_1 h_2}\cosh[Nh_1]\ - e^{-2\alpha_1 h_2}\cosh[Nh_1])\right]$$

$$a_{13} = \left[\left(\frac{M^2 B_r a_5}{\alpha_1^2 + N^2}\right)(e^{\alpha_1 h_2}\cosh[Nh_1]\ - e^{\alpha_1 h_2}\cosh[Nh_1])\right]$$

$$a_{14} = \left[\left(\frac{M^2 B_r a_6}{\alpha_1^2 + N^2}\right)(e^{-\alpha_1 h_2}\cosh[Nh_1]\ - e^{-\alpha_1 h_2}\cosh[Nh_1])\right]$$

$$a_{15} = \left[\left(\frac{M^2 B_r a_7}{N^2}\right)(\cosh[Nh_1]\ - \cosh[Nh_1])\right]$$

$$a_{16} = [\sinh[Nh_1]\cos[Nh_2] - \sinh[Nh_2]\cos[Nh_1]]$$

The coefficients of the heat transfer Zh_1 and Zh_2 at the walls $y = h_1$ and $y = h_2$ respectively, are given by

$$Zh_1 = \theta_y h_{1x} \quad (20)$$

$$Zh_2 = \theta_y h_{2x} \quad (21)$$

The solutions of the coefficient of heat transfer at $y = h_1$ and $y = h_2$ are given by

$$Zh_1 = \theta_y h_{1x} = \left(-a_9 N \sin[Ny] + a_8 N \cos[Ny] - \left[\frac{2\alpha_1 M^2 B_r a_3}{4\alpha_1^2 + N^2}\right] e^{2\alpha_1 y} + \left[\frac{2\alpha_1 M^2 B_r a_4}{4\alpha_1^2 + N^2}\right] e^{-2\alpha_1 y} - \left[\frac{\alpha_1 M^2 B_r a_5}{\alpha_1^2 + N^2}\right] e^{\alpha_1 y} + \left[\frac{\alpha_1 M^2 B_r a_6}{\alpha_1^2 + N^2}\right] e^{-\alpha_1 y}\right) (-2\pi\ cos[2\pi(x-t) + \emptyset] - k_1) \quad (22)$$

$$Zh_2 = \theta_y h_{2x} = \left(-a_9 N \sin[Ny] + a_8 N \cos[Ny] - \left[\frac{2\alpha_1 M^2 B_r a_3}{4\alpha_1^2 + N^2}\right] e^{2\alpha_1 y} + \left[\frac{2\alpha_1 M^2 B_r a_4}{4\alpha_1^2 + N^2}\right] e^{-2\alpha_1 y} - \left[\frac{\alpha_1 M^2 B_r a_5}{\alpha_1^2 + N^2}\right] e^{\alpha_1 y} + \left[\frac{\alpha_1 M^2 B_r a_6}{\alpha_1^2 + N^2}\right] e^{-\alpha_1 y}\right) (2\pi\varepsilon\ cos[2\pi(x-t)] + k_1) \quad (23)$$

The volumetric flow rate in the wave frame is defined by

$$q = \int_{h_1}^{h_2} (a_1 \sinh[\alpha_1 y] + \cosh[\alpha_1 y] + a)\, dy$$

$$\frac{a_1}{\alpha_1}[\cosh[\alpha_1 h_2] - \cosh[\alpha_1 h_1]] + \frac{a_2}{\alpha_1}[\sinh[\alpha_1 h_2] - \sinh[\alpha_1 h_1]] + [a(h_2 - h_1)] \quad (24)$$

The pressure gradient obtained from equation (24) can be expressed as

$$\frac{dp}{dx} = \left[\frac{M^2}{1+m^2} + \frac{1}{D}\right] - \left[\frac{q + BE - DF}{(h_2 - h_1) + DF - BE}\right]\left[\frac{M^2}{1+m^2} + \frac{1}{D}\right] \quad (25)$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q = \int_{h_2}^{h_1} (u+1)\, dy\ = q - h \quad (26)$$

The average volume flow rate over one wave period (T = λ/c) of the peristaltic wave is defined as

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, dt\ = q + 1 + d \quad (27)$$

From the equations (25) and (27), the pressure gradient can be expressed as

$$\frac{dp}{dx} = \left[\frac{M^2}{1+m^2} + \frac{1}{D}\right] - \left[\frac{(\bar{Q} - 1 - d) + BE - DF}{(h_2 - h_1) + DF - BE}\right]\left[\frac{M^2}{1+m^2} + \frac{1}{D}\right] \quad (28)$$

Where

$$B = \left[1 + \left[\frac{\cosh[\alpha_1 h_2]}{\left(\frac{\cosh[\alpha_1 h_1] - \cosh[\alpha_1 h_2]}{\sinh[\alpha_1 h_1] - \sinh[\alpha_1 h_2]}\right)\sinh[\alpha_1 h_1] - \cosh[\alpha_1 h_1]}\right]\right]$$

$$D = \left[\frac{1}{\left(\frac{\cosh[\alpha_1 h_1] - \cosh[\alpha_1 h_2]}{\sinh[\alpha_1 h_1] - \sinh[\alpha_1 h_2]}\right)\sinh[\alpha_1 h_1] - \cosh[\alpha_1 h_1]}\right]$$

$$E = \left[\frac{\cosh[\alpha_1 h_2] - \cosh[\alpha_1 h_1]}{\alpha_1 \sinh[\alpha_1 h_2]}\right] \qquad F = \left[\frac{\sinh[\alpha_1 h_2] - \sinh[\alpha_1 h_1]}{\alpha_1}\right]$$

4. DISCUSSION OF THE PROBLEM

The objective of this research is to study hall currents and joule heating on peristaltic blood flow in the porous medium through a vertical tapered asymmetric channel under influence of radiation. In order to find out numerical solutions, **MATHEMATICA** software is used.

Fig. 2 indicate the behaviour of axial velocity with y for different values of hall current parameter m with fixed Da = 0.1, M = 2, k_1= 0.1, dp/dx = -1, t = 0.4,∅ = π/6, x = 0.6, ε = 0.2. It is clear from the figure that the velocity gradually enhances by increase in Hall current parameter m.

The impact of porosity parameter Da on axial velocity distribution is shown in figure 3. We perceive from this graph that the axial velocity distribution gradually increases by increase in porosity parameter Da (Da = 0.1, 0.2, 0.3). Fig.4 depicts the axial velocity distribution (u) with dissimilar values of M (M = 2, 4, 6) with fixed m = 0.5, Da = 0.1, k_1= 0.1, dp/dx = -1, t = 0.4,∅ = π/6, x = 0.6, ε = 0.2. It can be notice from this graph that with an increase in magnetic field parameter M, the results in velocity profile diminished.

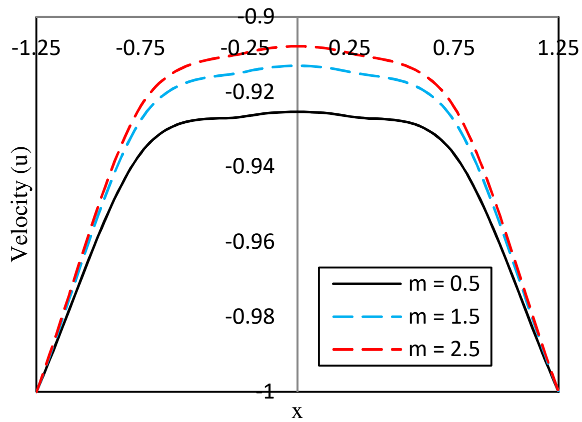

Fig. 2 Impact of m on axial velocity distribution with Da = 0.1, M = 2, k_1= 0.1, dp/dx = -1, t = 0.4,∅ = π/6, x = 0.6, ε = 0.2

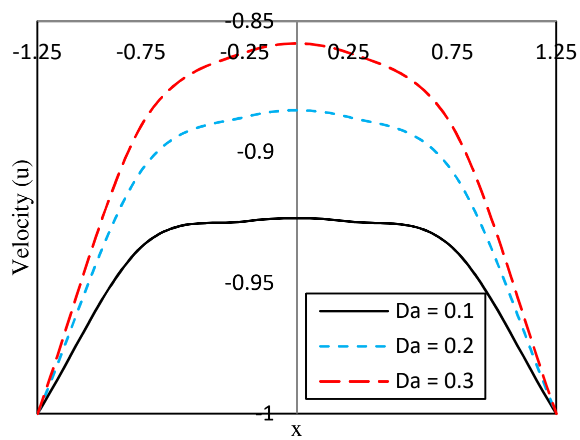

Fig. 3 Impact of Da on axial velocity distribution with m = 0.5, M = 2, k_1= 0.1, dp/dx = -1, t = 0.4,∅ = π/6, x = 0.6, ε = 0.2

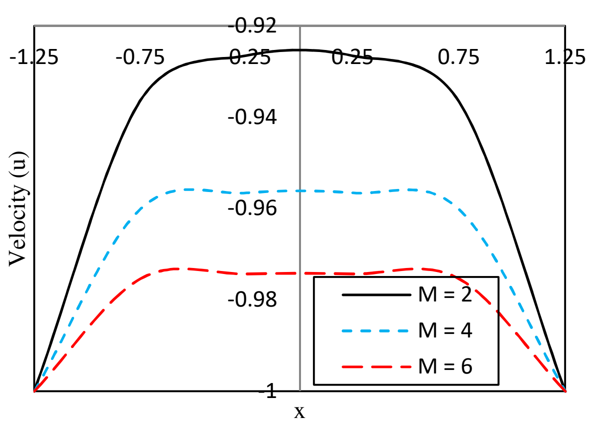

Fig. 4 Impact of M on axial velocity distribution with m = 0.5, k_1= 0.1 Da = 0.1, dp/dx = -1, t = 0.4,∅ = π/6, x = 0.6, ε = 0.2

Fig.5 describe the influence of different hall current parameters m on axial pressure gradient with fixed Da = 0.1, $\bar{Q}$ = 0.2, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, ∅ = π/6, ε = 0.2. We notice from this graph that the dp/dx reduces by increase in m (m = 0.5, 1.5, 2.5). Influence of magnetic field parameter M on pressure gradient depicted in figure 6. Indeed, the axial pressure gradient increases by increase in M (M = 2, 4, 6) with fixed other parameters.

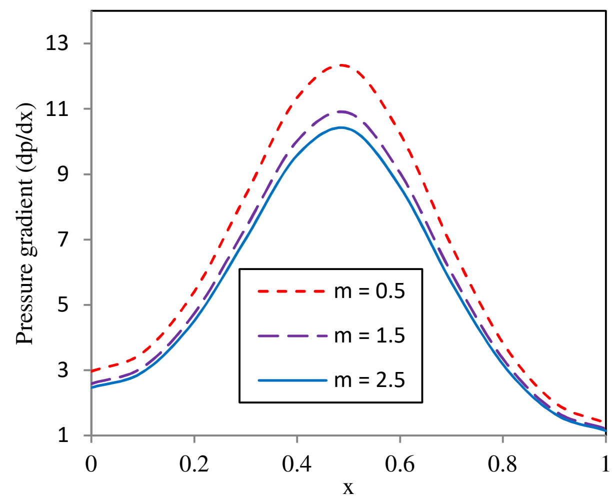

Fig. 5 Impact of m on $\frac{dp}{dx}$ with Da = 0.1,$\bar{Q}$ = 0.2, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, ∅ = π/6, ε = 0.2

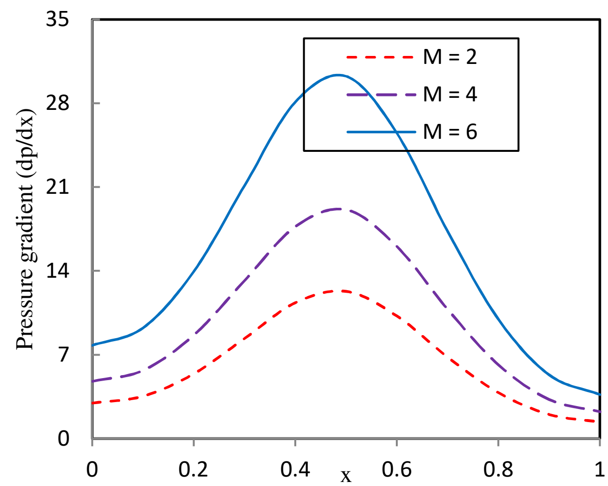

Fig. 6 Impact of M on $\frac{dp}{dx}$ with Da = 0.1,$\bar{Q}$ = 0.2, m = 0.5, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, ∅ = π/6, ε = 0.2

Fig. 7 reveals the dissimilar values of Da on dp/dx with fixed other parameters. It can be notice that the axial pressure gradient reduces by increase in Da. Figure 8 represents the flow structure of the axial pressure gradient for varied values of $\bar{Q}$ with fixed M = 2, Da = 0.1, m = 0.5, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, ∅ = π/6, ε = 0.2. It was observed that when $\bar{Q}$ increased, the result in axial pressure gradient reduces. An important result presented in figure 9.We observe form this graph that the dp/dx increases in the region $x\epsilon[0, 0.5]$ and the slowly reduces in the other region $x\epsilon[0.5, 1]$ by increase in φ.

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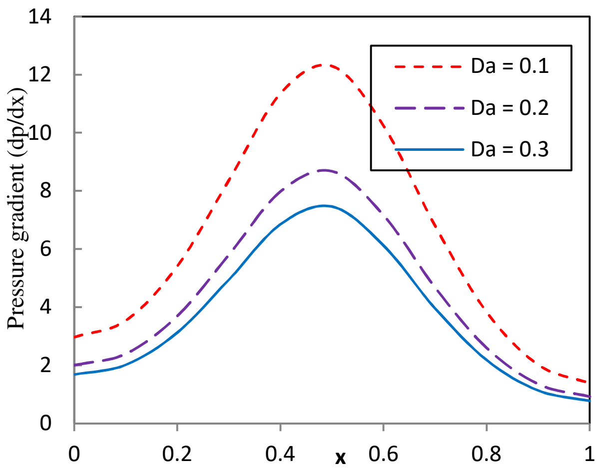

Fig. 7 Impact of Da on $\frac{dp}{dx}$ with M = 2, Q̄ = 0.2, m = 0.5, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, ∅ = π/6, ε = 0.2

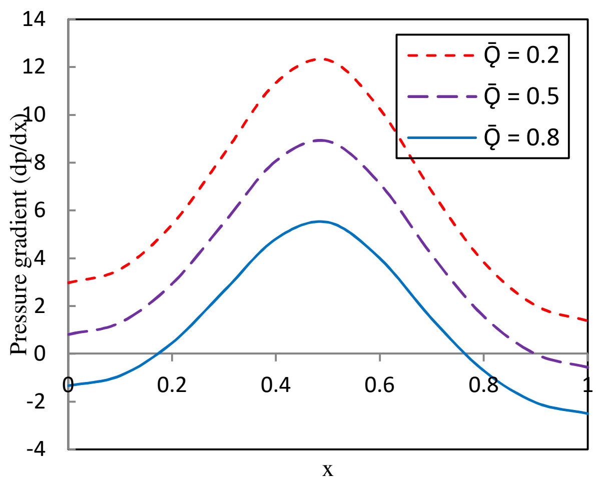

Fig. 8 Impact of Q̄ on $\frac{dp}{dx}$ with M = 2, Da = 0.1, m = 0.5, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, ∅ = π/6, ε = 0.2

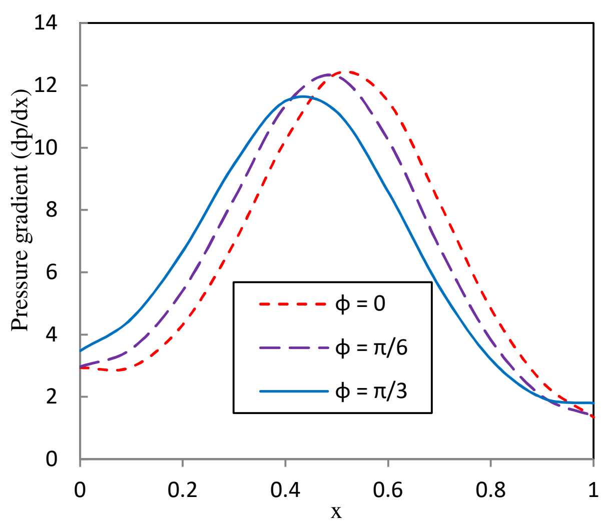

Fig. 9 Impact of ∅ on $\frac{dp}{dx}$ with M = 2, Da = 0.1, m = 0.5, t = $\frac{\pi}{4}$, M = 2, k_1= 0.1, d = 2, Q̄ = 0.2, ε = 0.2

Fig.10 presents the various values of m (m = 0.5, 1.5, 2.5) on temperature profile (θ). It has been inferred that the temperature distribution diminished by increase in m with fixed Da = 0.3, N = 0.4, Pr =1.5, M = 2, β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2. Influence of M on temperature distribution (θ) is displayed in fig.11 with fixed other parameters. We perceive from this graph that the increase in M, the results in the flow field rises. Fig.12 shows to examine the effect of Da on temperature distribution with fixed M = 2, N = 0.4, Pr =1.5, m = 0.5, β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2. This graph indicates that the results in the temperature profile reduce by increase in Da.

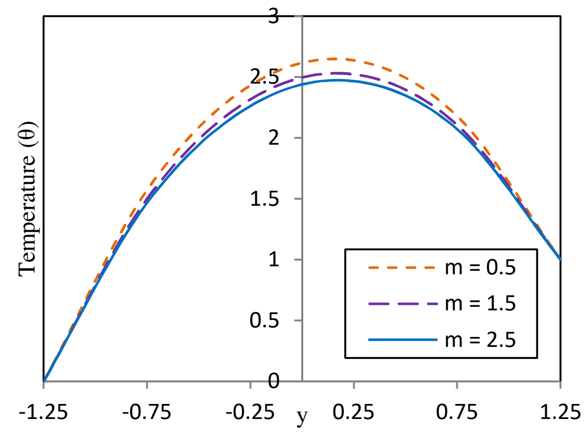

Fig. 10 Impact of m on θ with Da = 0.3, N = 0.4, Pr =1.5, M = 2 β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

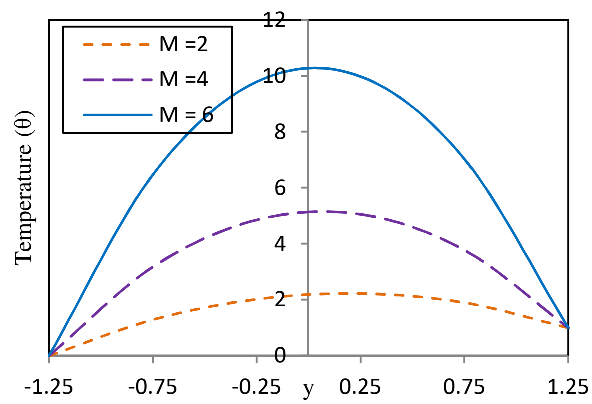

Fig. 11 Impact of M on θ with Da = 0.1, N = 0.4, Pr =1.5, m = 0.5, β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

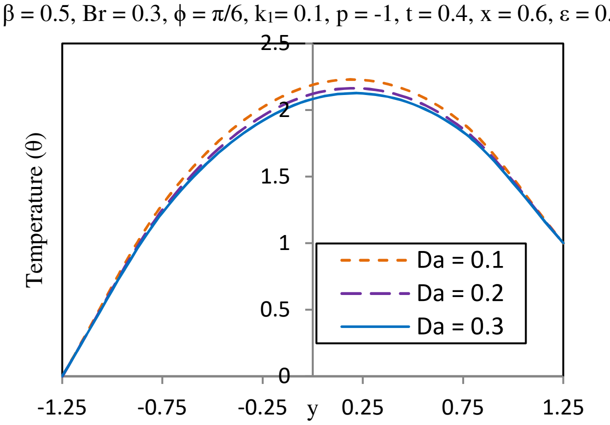

Fig. 12 Impact of Da on θ with M = 2, N = 0.4, Pr =1.5, m = 0.5, β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

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Fig.13 shows the temperature distribution with various values of Radiation parameter N with fixed other parameters. The results in temperature distribution rises by rise in N. Fig.14 depicts the temperature distribution (θ) with dissimilar values of Pr (Pr = 1.5, 3, 4.5) with fixed other parameters. It can be observed that from this graph that the temperature distribution enhances by increase in Pr. Effect of Brinkman number Br on temperature of the fluid (θ) displayed in fig.15. It clear from this figure that the temperature of the fluid rises by increase in Br (Br = 0.3, 0.5, 0.7).

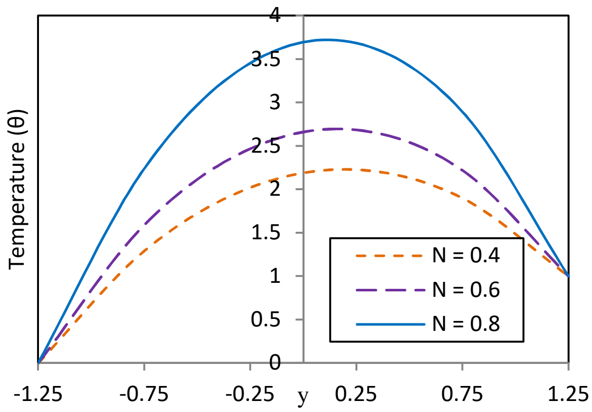

Fig. 13 Impact of N on θ with M = 2, Da = 0.1, Pr =1.5, m = 0.5, β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

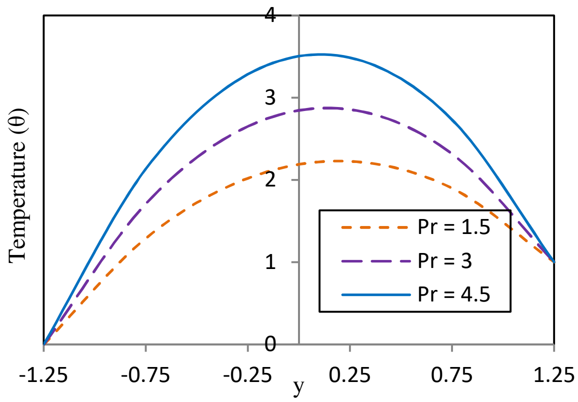

Fig. 14 Impact of Pr on θ with M = 2, Da = 0.1, N = 0.4, m = 0.5, β = 0.5, Br = 0.3, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

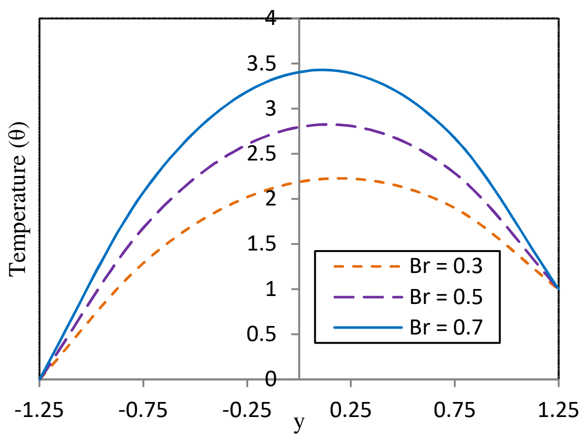

Fig. 15 Impact of Br on θ with M = 2, Da = 0.1, N = 0.4, m = 0.5, β = 0.5, Pr = 1.5, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

An influence of magnetic field parameter M on Z (coefficient of heat transfer) at y = h_1 wall is shown in figure 16 with fixed other parameters. It clear from this figure that with increase in M (M = 2, 4, 6), the fluid in the coefficient of heat transfer rises in x ∈ [0, 0.04] U [0.58, 1] and it reduces in the other region x ∈ [0.04, 0.58]. Figure 17 reveals the heat transfer coefficient with radiation parameter being fixed other parameters. It can be seen that with rise in N (N = 0.4, 0.6, 0.8), the results in Z enhances in the region x ∈ [0, 0.04] U [0.58, 1] and the fluid in Z reduces in the other region x ∈ [0.04, 0.58]. Effect of Pr on Z at y = $h_1$wall is shown in figure 18. It observed that with the rise in Pr, the coefficient of heat transfer distribution (Z) enhances in the region x ∈ [0, 0.04] U [0.58, 1] whereas the results in Z reduces in the other region x ∈ [0.04, 0.58].

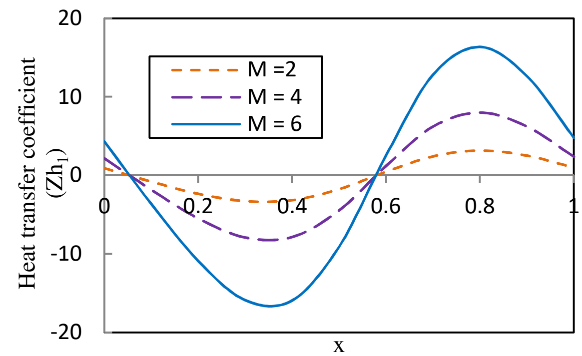

Fig. 16 Influence of M on coefficient of heat transfer with m = 0.5, Da = 0.1, N = 0.4, Br = 0.3, β = 0.5, Pr = 1.5, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

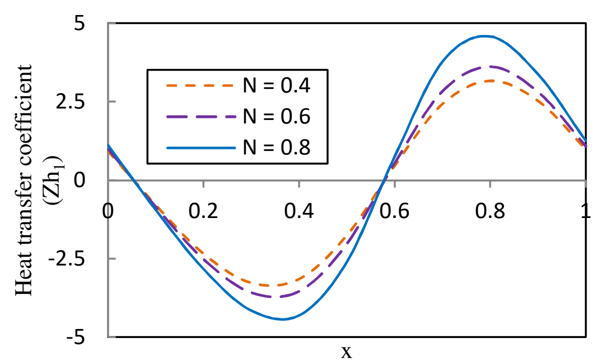

Fig. 17 Influence of N on coefficient of heat transfer with m = 0.5, Da = 0.1, M = 2, Br = 0.3, β = 0.5, Pr = 1.5, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

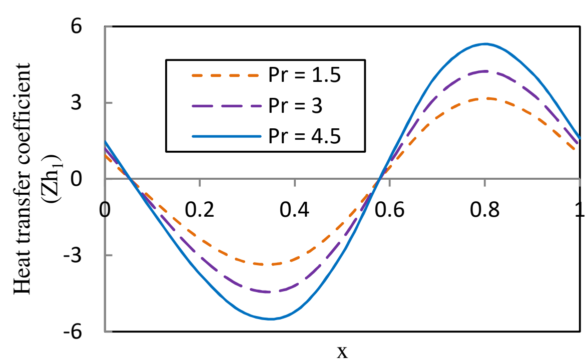

Fig. 18 Influence of Pr on coefficient of heat transfer with m = 0.5, Da = 0.1, M = 2, Br = 0.3, β = 0.5, N = 0.4, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

Figure 19 presents the heat transfer coefficient with Br. It was evident that from the graph that with rise Br, the results in Z enhances in the region x ∈ [0, 0.04] U [0.58, 1] whereas the results in Z slowly reduces in the other region x ∈ [0.04, 0.58]. An impact of hall current parameter m on Z (coefficient of heat transfer) at $y = h_1$ wall is shown in figure 20 with fixed other parameters. It is interesting to note that the result in coefficient of heat transfer coefficient distribution (Z) reduces in the region x ∈ [0, 0.04] U [0.58, 1] and the results in the other region x ∈ [0.04, 0.58] enhances with increase in m.

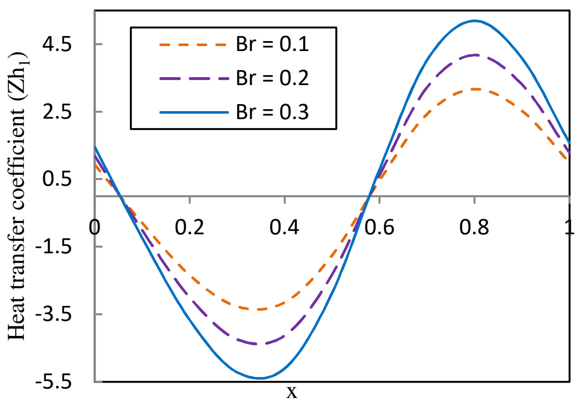

Fig. 19 Influence of Br on coefficient of heat transfer with m = 0.5, Da = 0.1, M = 2, Pr = 1.5, β = 0.5, N = 0.4, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

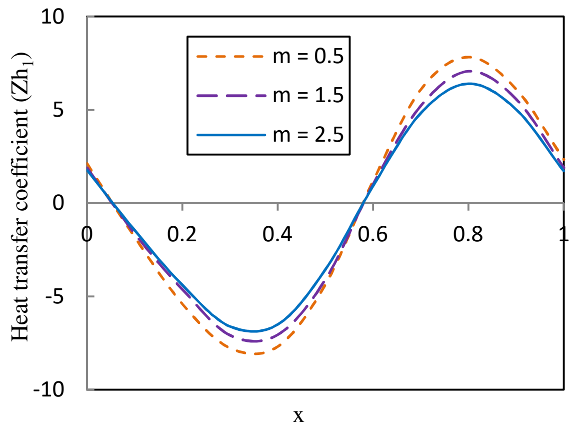

Fig. 20 Influence of m on coefficient of heat transfer with M = 4, Da = 0.3, N = 0.4, Br = 0.3, β = 0.5, Pr = 1.5, ϕ = π/6, k_1= 0.1, p = -1, t = 0.4, x = 0.6, ε = 0.2

5. CONCLUSIONS

Hall currents and joule heating on hemodynamic peristaltic flow with porous medium through a vertical tapered asymmetric channel under the influence of radiation-Blood flow analysis model have been studied through this paper. The mathematical problem is solved analytically by using long wavelength and low Reynolds number assumptions. The significant findings of this paper are summarized below.

1. The axial velocity distribution rises with rise in hall current parameter (m) and porous parameter (Da) and the axial velocity distribution reduces when magnetic field parameter (M) increased.
2. When rise in hall current parameter (m), porous parameter (Da) and volumetric flow rate ($\bar{Q}$), the results in axial pressure gradient diminished.
3. The temperature of the fluid increases when Magnetic field parameter (M), Radiation parameter (N), Prandtl number (Pr) and Brinkman number (Br) increased whereas the results in temperature of the fluid reduces when increase in hall current parameter (m) and porous parameter (Da) increased.
4. Heat transfer coefficient rises in x ∈ [0, 0.04] U [0.58, 1] and it reduces in the other region x ∈ [0.04, 0.58] with increase in M, N, Pr and Br.
5. Heat transfer coefficient distribution (Z) reduces in the region x ∈ [0, 0.04] U [0.58, 1] and the results in the other region x ∈ [0.04, 0.58] enhances with increase in hall currents parameter m.

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