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SORET AND DUFOUR EFFECTS ON MHD RADIATIVE HEAT AND MASS TRANSFER FLOW OF A JEFFREY FLUID OVER A STRETCHING SHEET

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ABSTRACT

This paper studies the combined effects of Soret (thermal-diffusion) and Dufour (diffusion-thermo) on magnetohydrodynamics (MHD) boundary layer flow of a Jeffrey fluid past a stretching surface with chemical reaction and heat source. Using the similarity transformations, the governing equations are transformed into a set of non-linear ordinary differential equations (ODE's). The resulting equations are then solved numerically by using the shooting method along with Runge-Kutta fourth order integration scheme. Numerical results for the velocity, temperature and concentration distributions as well as the skin-friction coefficient, Nusselt number and Sherwood number are discussed in detail and displayed graphically for various physical parameters. The results indicate that the influence of Soret and Dufour numbers are significantly active in the study of non-Newtonian fluid flows. The accuracy of the numerical method is tested by comparing with previously published work as a limiting case (for viscous flow) and the results are found to be in excellent agreement.

Keywords: *Soret and Dufour, MHD, Jeffrey fluid, Stretching sheet, Chemical reaction, Thermal radiation.*

1. INTRODUCTION

In the recent years, boundary layer flow of non-Newtonian fluids over a stretching surface has received special attention from the researchers (Ellahi *et al.* 2012; Nadeem *et al.,* 2014; Bose *et al.*, 2015; Akbar *et al.,* 2016; Sahoo, 2010). This is because of their advanced industrial, engineering science and technological applications such as glass fibber, wire drawing, paper production, extrusion of plastic sheet, hot rolling, drawing of plastic films and many others. Motivated by these facts, Qasim (2013) examined the heat and mass transfer of an incompressible viscous Jeffrey fluid flow over a stretching surface in the presence of heat source/sink. Hayat *et al.* (2014) investigated the unsteady boundary layer flow of an incompressible non-Newtonian Jeffrey fluid over a stretching sheet by using HAM. Ramesh (2015) addressed the effect of heat source on stagnation point flow over a stretching surface with a Jeffrey nano-liquid. Convective radiative flow a Jeffrey fluid over an inclined stretching cylinder has been studied by Hayat *et al.* (2015). Ramachandra Prasad *et al.* (2015) analyzed the heat and mass transfer of an incompressible Jeffrey non-Newtonian fluid past a vertical porous plate. Recently, the flow and heat transfer of a nanofluid in a rotating system with first order chemical reaction is discussed by Venkateswarlu and Satya Narayana (2015). Further, several researchers have explored the flow behaviour due to the non-Newtonian phenomenon in various configurations (Sheikholeslami *et al.,* 2014; Hayat *et al.,* 2015; Jail *et al.,* 2013; Rashidi *et al.,* 2015).

The study of the hydrodynamic flow of an electrically conducting non-Newtonian fluid over stretching surface is motivated by its great values in engineering problems such as plasma studies, geothermal energy extraction, cooling of nuclear reactors and many other fields. In view of these applications, Sigey *et al.* (2013) studied the MHD free convection flow past a vertical porous plate with Joule heating. Akram and Nadeem (2013) examined the effect of induced magnetic field and heat transfer on the peristaltic motion of a Jeffrey fluid in an asymmetric channel. Shehzad *et al.* (2014) investigated the MHD three-dimensional flow of Jeffrey fluid with Joule heating. Ellahi and Hussain (2014) studied the partial slip effect on MHD peristaltic flow of a Jeffery fluid in a rectangular duct. Recently, Rashidi *et al.* (2015) presented the free convective heat and mass transfer of MHD fluid flow over a vertical stretching porous sheet with radiation. Ellahi *et al.* (2013) examined the series solutions of MHD peristaltic flow of a Jeffrey fluid in eccentric cylinders. Satya Narayana *et al.* (2016) considered the effects of thermal radiation on MHD heat and mass transfer of a Jeffrey fluid due to stretching sheet with chemical reaction. Sheikholeslami *et al.* (2015) investigated the effect of space dependent magnetic field on ferrofluid flow and heat transfer with free convection of Fe_3O_4–water nanofluid. Interesting investigations on MHD flows can be seen in the references (Chamkha *et al.*, 2014; Ibrahim *et al.*, 2013; Ellahi *et al.*, 2014; Harish Babu and Satya Narayana, 2016).

It is also important to consider Soret and Dufour effects on heat and mass transfer of a non-Newtonian fluid over a stretching sheet. These play a very significant role in many industrial and practical applications such as the operation of solar ponds, the microstructure of the world oceans, biological systems and petrology on chemical engineering etc. Soret (1880) was the first who introduced the Soret effect in a tube at two ends. A diffusion-thermo (Dufour) effect is energy flux can be generated by temperature gradient as well as composition gradient. Postelnicu (2007) studied the Soret and Dufour effects over a vertical surface in the presence of convection and chemical reaction. Beg *et al.* (2009) considered the Soret and Dufour effects on MHD heat and mass transfer of a saturated flow over a

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permeable stretching sheet. Pal *et al.* (2013) studied the heat and mass transfer characteristics of a fluid over a non- isothermal wedge with Ohmic dissipation. Hayat *et al.* (2012) analyzed the Soret and Dufour effects on MHD flow of a Casson fluid over a stretching surface. Soret and Dufour effects on the stagnation point flow of Jeffery fluid with convective boundary condition is investigated by Shehzad *et al.* (2013). Rashidi *et al.* (2015) examined the heat and mass transfer of a viscoelastic fluid flow over a vertical stretching sheet with Soret and Dufour effects using HAM. Hayat *et al.* (2016) discussed the radial magnetic field of peristaltic transport in a curved channel with Soret and Dufour effects. Many investigations were made to examine flow over various flow fields under different aspects (Goyal *et al.,* 2014; Venkateswarlu *et al.,* 2015; Hayat *et al.,* 2015; Satya Narayana, 2015; Sheikholeslami *et al.,* 2015).

The physical situation defined in all the above investigations is connected to the process of uniform temperature and concentration over Newtonian fluid flows (Kumar, 2009; Ali, 1995; Elbashbeshy, 1998; Andersson et al. 1992). Whereas, the influence of power law form of temperature and concentration on non-Newtonian fluid flows has received a little attention even though it has huge applications in many industrial and engineering sciences. This motivates the present work to explore the Soret and Dufour effects on MHD electrically conducting non-Newtonian Jeffrey fluid over a linear stretching sheet in the presence of chemical reaction and heat source. After using similarity transformations, the governing equations are converted to a system of non-linear ordinary differential equations. The resulting equations are then solved numerically by using fourth order Runge-Kutta method along with shooting technique. The effect of different parameters on velocity, temperature and concentration profiles are shown with the help of graphs and tables. Further, the skin friction coefficient, Nusselt and Sherwood numbers are computed and analyzed.

2. FORMULATION OF THE PROBLEM

We consider the steady two-dimensional flow of an incompressible, electrically conducting Jeffery fluid past a stretching sheet in the presence of Soret and Dufour effects. The sheet is stretched with linear velocity ($U_w(x)=cx$) and the free stream velocity are assumed to proportional with the distance $x-$ from the origin (see Fig.1). The flow is confined to $y>0$. The surface of the sheet is also assumed to be subjected to power law form of temperature ($T_w = T_\infty + A_1(x/l)^m$) and concentration ($C_w = C_\infty + A_2(x/l)^m$). Where T_∞ and C_∞ are the ambient fluid temperature and concentration respectively, $l, m, A_1 = A_2 > 0$ are constants. Further, a uniform strength of magnetic field B_0 is applied transversally to the direction of the flow. The magnetic Reynolds number is assumed to be small and thus the induced magnetic field is negligible. It is also assumed that Dufour effect may be defined by a second order concentration derivative with respect to the oblique coordinate in the energy equation whereas Soret effect is defined by the second order temperature derivative in the mass diffusion equation. The fundamental equations for Jeffrey fluid can be written as (Nadeem *et al.,* (2009))

$$\tau = -pI + S$$

$$S = \frac{\mu}{1+\lambda}\left\{R_1 + \lambda_1\left(\frac{\partial R_1}{\partial t} + V.\nabla\right)R_1\right\}$$

The Rivlin-Ericksen tensor defined by

$$R_1 = (\nabla V) + (\nabla V)'$$

Under these assumptions, the governing boundary layer equations are (see Refs. Hayat *et al.,* 2014; Hayat *et al.,* 2015; Satya Narayana and Harish Babu, 2016)

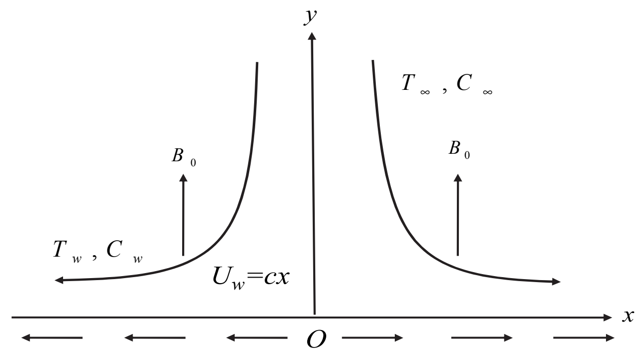

Fig. 1 Schematic diagram of Jeffrey fluid flow over a stretching Surface

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\upsilon}{1+\lambda}\left\{\frac{\partial^2 u}{\partial y^2} + \lambda_1\left(\begin{matrix} u\dfrac{\partial^3 u}{\partial x \partial y^2} + v\dfrac{\partial^3 u}{\partial y^3} \\ -\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x \partial y}\end{matrix}\right)\right\} - \frac{\sigma B_0^{\,2}}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_P}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_P}\frac{\partial q_r}{\partial y} - \frac{Q}{\rho c_P}(T - T_\infty) + \frac{Dk_T}{c_P c_S}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - Kr^*(C - C_\infty) + \frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The following boundary conditions are appropriate in order to employ the effect of stretching of the boundary surface may be written as

$$u = U_W(x), v = 0, T = T_w, C = C_W \quad at \quad y = 0$$
$$u \to 0, \quad u' \to 0, \quad T \to T_\infty, C \to C_\infty \;\; as \;\; y \to \infty \tag{5}$$

Employing the Roseland diffusion approximation the radiative heat flux q_r is given by $q_r = -\dfrac{4\sigma^*}{3K_s}\dfrac{\partial T^4}{\partial y}$ (6)

where K_s and σ^* is the Roseland mean absorption coefficient and the Stefan-Boltzman constant respectively.

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature, we can expand T^4 in Taylor's series about T_∞ and neglecting the higher order terms beyond the first degree (T- T_∞), we get

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

With the help of equations (6) and (7), equations (3) can be written as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_P}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_P}\frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} - \frac{Q}{\rho c_P}(T - T_\infty) + \frac{Dk_T}{c_P c_S}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

The following similarity transformations are used to transform the boundary layer flow heat and mass transfer equations to non-linear ordinary differential equations

$$\eta = \sqrt{\frac{c}{\upsilon}}y, u = cxf'(\eta), v = -\sqrt{c\upsilon}f(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_W - C_\infty} \tag{9}$$

where η is the similarity variable and $f(\eta)$ is the dimensionless stream function and f', θ, ϕ respectively, are the dimensionless velocity, temperature and concentration.

Using the similarity transformations of equation (9), equations (2), (4) and (8) are transformed to the following ordinary differential equations

$$f''' + (1+\lambda)(ff'' - f'^2) + \beta(f''^2 - ff'''') - (1+\lambda)Mf' = 0 \quad (10)$$

$$\left(1+\frac{4}{3}R\right)\theta'' + \Pr f\theta' - \Pr(mf' + \gamma)\theta + D_f \Pr \phi'' = 0 \quad (11)$$

$$\phi'' + Scf\phi' - Sc(mf' - Kr)\phi + ScSr\theta'' = 0 \quad (12)$$

Where $\beta = \lambda_1 c$ is the Deborah number and $M = \frac{\sigma B_0^2}{\rho c}$ is the Hartmann number, $R = \frac{4\sigma^* T_\infty^3}{K_s k}$ is the radiation parameter, $\Pr = \frac{\rho c_p}{k}$ is the Prandtl number and $\gamma = \frac{Q\upsilon}{\rho c_p}$ is the heat source/sink parameter, $Sc = \frac{\upsilon}{D}$ is the Schmidt number and $Kr = \frac{Kr^* \delta^2}{\nu}$ is the chemical reaction parameter, $D_f = \frac{Dk_T(C_w - C_\infty)}{c_s c_p \upsilon(T_w - T_\infty)}$ is the Dufour number, $Sr = \frac{Dk_T(T_w - T_\infty)}{\upsilon T_m (C_w - C_\infty)}$ is the soret number.

In view of the transformations, Eq. (5) takes the following non-dimensional form

$$\begin{aligned} &f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \phi(\eta) = 1 \quad at \quad \eta = 0 \\ &f'(\eta) = 0, f''(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 \quad as \quad \eta \to \infty \end{aligned} \quad (13)$$

The most important physical quantities for the problem are skin-friction coefficient C_f, local Nusselt number Nu and Sherwood number Sh which are defined by the following relations:

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2}, \; Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \; Sh_x = \frac{xm_w}{D(C_w - C_\infty)} \quad (14)$$

Where, k is the thermal conductivity of the fluid.

The skin friction on the sheet τ_w, rate of heat transfer q_w, D is the mass diffusivity, and the rate of mass transfer m_w are given by

$$\tau_w = \frac{1+\beta}{1+\lambda}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \; q_w = \left(1+\frac{4}{3}R\right)\left(\frac{\partial T}{\partial y}\right)_{y=0}, \; m_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} \quad (15)$$

Substituting eq.(9) in eq.(14) using eq.(15), we get

$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{1+\lambda}\{f''(0) + \beta f'''(0)\}, \; Nu_x \mathrm{Re}_x^{-1/2} = -\left(1+\frac{4}{3}R\right)\theta'(0),$$

$$Sh\,\mathrm{Re}_x^{-1/2} = -\phi'(0) \quad (16)$$

where $\mathrm{Re}_x = \frac{xu_w}{\upsilon}$ is the local Reynolds number.

3. NUMERICAL PROCEDURE

The coupled non-linear ODE's (10)–(12) that are subject to the boundary conditions (13) have been solved numerically using shooting method. In this method, the fourth order non-linear Eq. (10) and second-order Eqs. (11)-(12) have been reduced to eight simultaneously first order ordinary differential equations for which only eight unknowns following the method of superposition (Na, (1979). To solve this system we require eight initial conditions. Thus we employ numerical shooting technique with Runge–Kutta scheme. In order to determine η_∞ for the boundary value problem stated by equations (10)-(12), we start with some initial guess value for some particular set of physical parameters to obtain $f''(0)$. The solution procedure is repeated with another large value of η_∞ until two successive values of $f''(0)$ differing only by the specified significant digit. The last valve of η_∞ is finally chosen to be the most appropriate value of the limit $\eta \to \infty$ for the particular set of parameters. The value of η may change for another set of physical parameters. Once the finite value of η is determined then the coupled boundary value problem given by equations (10)-(12) are solved numerically using the shooting method.

4. RESULT AND DISCUSSION

The present work focuses on MHD radiative heat and mass transfer of a Jeffrey fluid over a stretching sheet in the presence of Soret and Dufour effects. The velocity, temperature, concentration, local skin friction coefficient, Nusselt and Sherwood number profiles for different parameters are displayed in Figs.2-16. In the present study we have chosen *M=0.2, m=2.0, R=0.1,* D_f *=0.3, Kr=0.2, Pr=0.72, Sc=0.3, Sr=0.2,* β *=1.0,* γ *=0.1* and λ *=1.0.* The accuracy of the present numerical solution is validated by comparing the present results with those of Grubka and Bobba (1985) and Chen (1998) for the viscous case. The numerical results are in good agreement with those obtained numerically as shown in Table 1. It can be observed that, in the absence of Soret and Dufour numbers, the present problem reduces to those of Satya Narayana and Harish Babu (2016).It is also noticed that, the present problem reduces to a regular viscous fluid (see Ref. Chen (1998)) if we choose $\beta = \lambda$ =0 in Eq. (10).

Table 1 Heat transfer coefficient $-\theta'(0)$ for various values of *Pr* when *m*=2.

Pr	Grubka and Bobba (1985)	Chen (1998)	Present
1.0	1.333	1.33334	1.33333
3.0	2.5097	1.50972	1.50972
10.0	4.7969	4.79686	4.79673

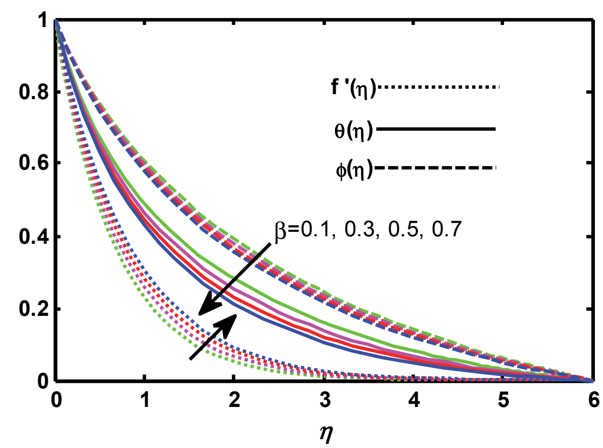

Fig. 2 Effect of β on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ profiles

Fig. 2 depicts the velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ profiles for various values of Deborah number β. It is noticed that, $f'(\eta)$ increases with the increase of β. From the definition of Deborah number, one can see that β is directly proportional to the rate of stretching sheet. Hence, the larger β has higher fluid motion in the boundary layer which in turn raises the fluid velocity. On the other hand, $\theta(\eta)$ and $\phi(\eta)$ decrease for larger values of β. Physically, β is proportional to retardation time and hence retardation time increased when β increases. Hence, an increase in retardation time corresponding to the lower temperature and weaker thermal boundary layer thickness.

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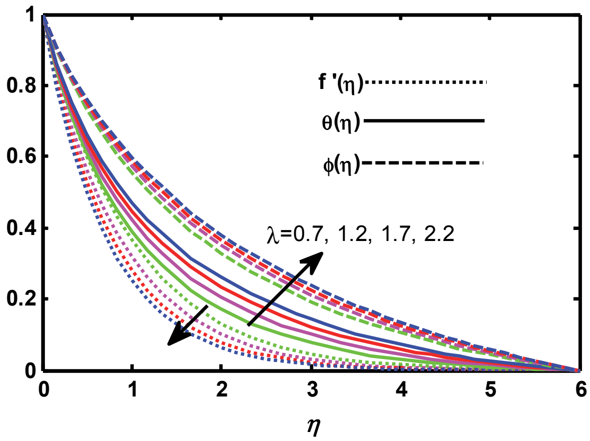

Fig. 3 Effect of λ on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ profiles

The influence of the ratio of relaxation to retardation time's parameter λ on the velocity, temperature and concentration distributions is shown in Fig. 3. The effect of increasing values of λ is to reduce the velocity profile and the boundary layer thickness. Conversely, $\theta(\eta)$ and $\phi(\eta)$ are enhanced with an increase of λ. It is also observed that λ and β has opposite effects on velocity, temperature and concentration profiles.

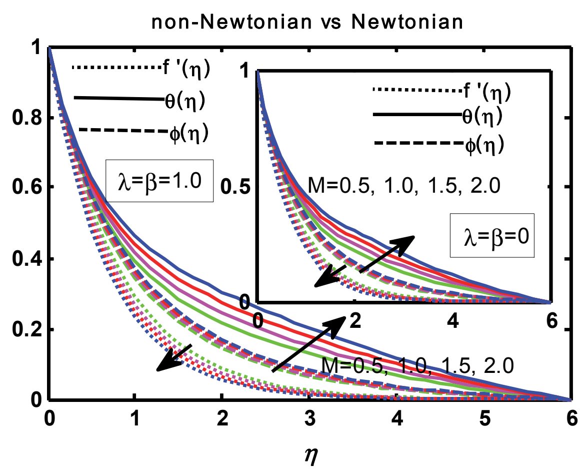

Fig. 4 Effect of M on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ profiles

Fig. 4 illustrates the effect of magnetic field parameter M on the velocity, temperature and concentration profiles. It is observed that $f'(\eta)$ decreasing behaviour for increasing values of M. Physically, the application of normal magnetic field has the propensity to give rise to a resistive type force called the Lorentz force and hence results in impeding the velocity profile and therefore, decreases the momentum boundary layer. On the other hand, both $\theta(\eta)$ and $\phi(\eta)$ increase with increasing values of M. Hence, the magnetic field tends to retard the velocity field which in turn induces the temperature and concentration fields increases for larger values of M. These results are same in both Newtonian and non-Newtonian cases and hence obviously maintained from the physical point of view.

The effects of Soret and Dufour numbers on $\theta(\eta)$ and $\phi(\eta)$ distributions across the boundary layer are shown in Figs. 5 and 6 respectively. It is clear from Fig. 5 that, the temperature distribution across the thermal boundary layer thickness reduces with increase of Sr. The reason behind this phenomenon is that, higher values of Sr reduces the thermal diffusivity, while opposite behavior can be observed for concentration distribution with the increasing values of Sr. Further, it is witnessed from Fig. 6 that both the temperature and concentration fields decrease with the increase of D_f along the surface. Physically, higher values of D_f produces the combined effects of thermal and solutal buoyancy forces, enhance convection velocity which is, in turn, leads to decreasing the temperature and concentration of the fluid. It is also noticed that the impact of D_f on concentration profiles is very less as compared to the temperature profiles.

It is observed from Fig. 7 that both $\theta(\eta)$ and $\phi(\eta)$ decrease with the increase of m. Physically, the fluid flow is caused by stretching sheet temperature and stretching sheet temperature is greater than the free stream temperature (i.e. $T_w > T_\infty$) and hence the temperature decreases with the increase of m. The influence of Schmidt number (*Sc)* on temperature and concentration profiles is shown in Fig. 8. It is observed that both $\theta(\eta)$ and $\phi(\eta)$ decrease with increase of Sc. Physically, an increase of Sc means decreases of molecular diffusion and hence $\phi(\eta)$ is higher for smaller values of Sc and lower for larger values of Sc.

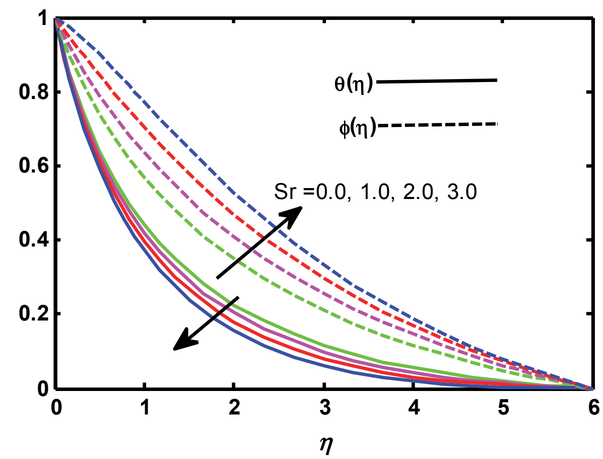

Fig. 5 Effect of Sr on $\theta(\eta)$ and $\phi(\eta)$ profiles

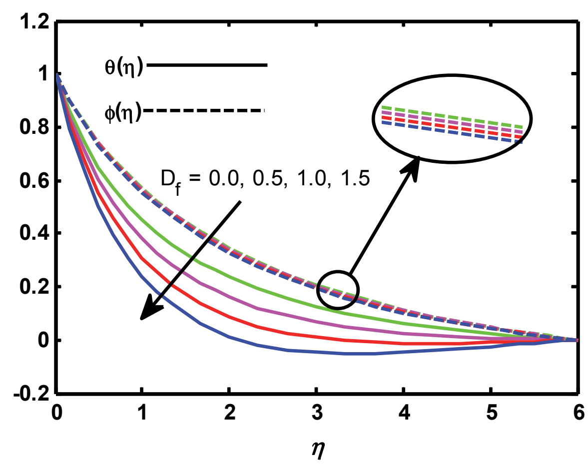

Fig. 6 Effect of D_f on $\theta(\eta)$ and $\phi(\eta)$ profiles

The influence of heat source (γ >0) / sink (γ <0) parameter on temperature and concentration distributions is highlighted respectively in Figs. 9(a) and 9(b). It can easily be seen that $\theta(\eta)$ increases in positive values γ and decreases for negative values γ. Physically, an increase of heat source in the boundary layer generates energy which causes the temperature of the fluid to increase and whereas heat sinks provide a decrease in the temperature of the fluid. Thus, the presence of

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heat sink in the boundary layer absorbs energy which results in the temperature of the fluid to decrease and hence heat sink is better suited for effective cooling of stretching sheet. Further, an opposite trend is observed in the case of concentration profile for different values of heat source/sink parameter γ.

Figure 10 shows the influence of chemical reaction parameter *Kr* on $\theta(\eta)$ and $\phi(\eta)$. It is obvious that the behaviour of increasing values of *Kr* produces a decrease in the temperature and concentration distributions. Thus, the bigger *Kr*, results in the steeper curves in the temperature and concentration boundary layer and hence this result shows the thinner boundary layer thickness and weaker molecular diffusivity.

Figure 11 illustrates the variation of dimensionless temperature and concentration profiles for different values of the thermal radiation parameter *R*. It is noticed that, $\theta(\eta)$ increases with increasing values of *R* and opposite trend is seen in the case of $\phi(\eta)$. This is due to the fact that thermal boundary layer thickness and the flux of energy transport to the fluid temperature increases with the increase of *R*.

Figure 12 shows the influence of Prandtl number *Pr* on temperature and concentration profiles. It is obvious that the behaviour of increasing values of *Pr* produces a decrease in the temperature distributions. Physically, larger Prandtl number corresponds to weaker thermal diffusivity which tends to the lower temperature and thinner thermal boundary layer thickness. Consequently, opposite behavior is observed in the case of $\phi(\eta)$.

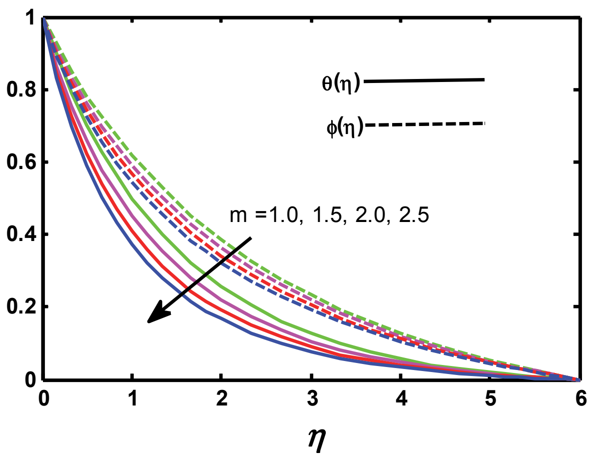


Fig. 7 Effect of *m* on $\theta(\eta)$ and $\phi(\eta)$ profiles

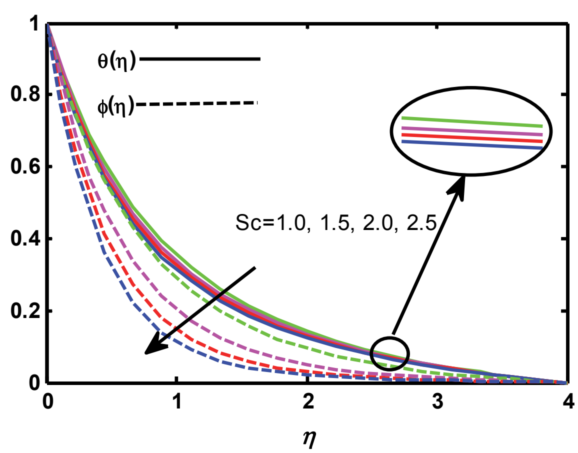
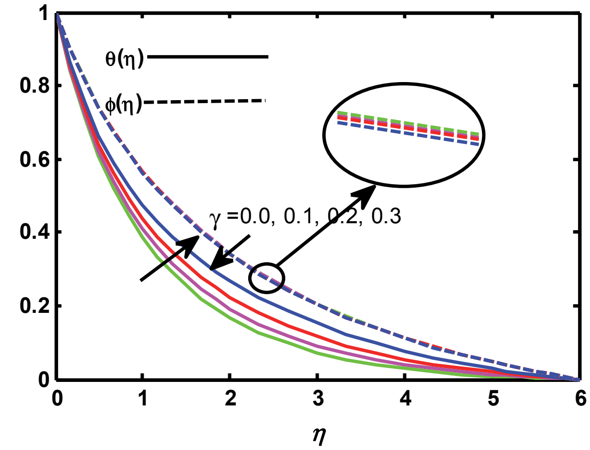
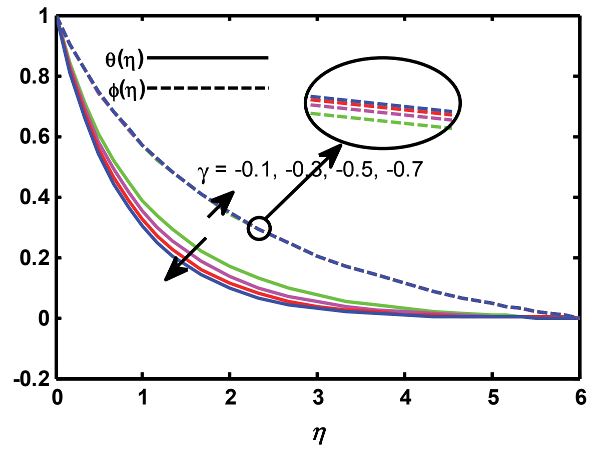


Fig. 8 Effect of *Sc* on $\theta(\eta)$ and $\phi(\eta)$ profiles



(a) $\gamma > 0$



(b) $\gamma < 0$

Fig. 9 Variation of $\theta(\eta)$ and $\phi(\eta)$ at a) heat source b) heat sink

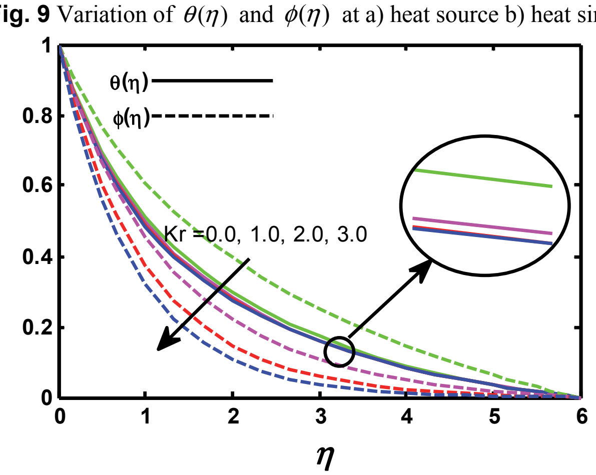


Fig. 10 Effect of *Kr* on $\theta(\eta)$ and $\phi(\eta)$ profiles

Figure 13 exhibits the skin friction coefficient $f''(0)$ against λ for different values of Deborah number β. It is noticed that $f''(0)$ decreases with the increase of β. This is due to the fact that the higher values of β lead to increasing movement of fluid particles in the boundary layer. Hence, the boundary layer thickness decreases which results in lower values of $f''(0)$. On the other hand, the same behavior can be observed for $f''(0)$ with increase of λ. This is an agreement with the physical realities that the $f''(0)$ remains high as compared to the case of Ref. [22] (i.e. D_f=0, *Sr*=0). Hence, it is understood that the influence of Soret and Dufour numbers are greatly effective in the study of non- Newtonian fluid flows over a stretching sheet.

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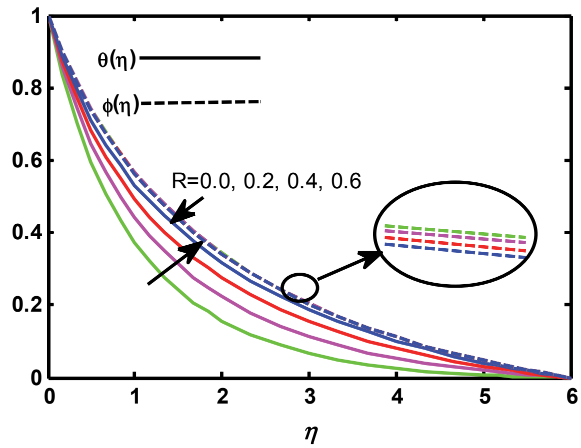


Fig. 11 Effect of R on $\theta(\eta)$ and $\phi(\eta)$ profiles

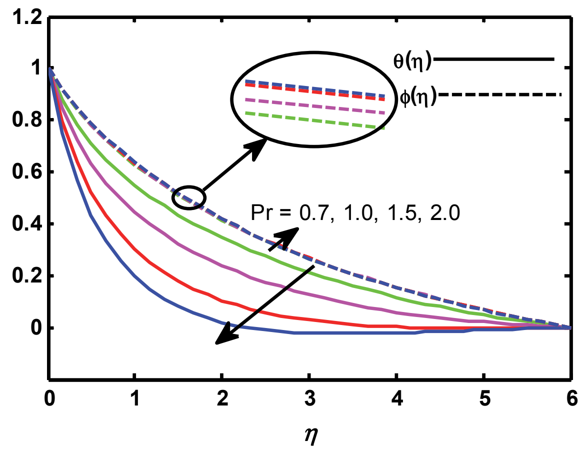


Fig. 12 Effect of Pr on $\theta(\eta)$ and $\phi(\eta)$ profiles

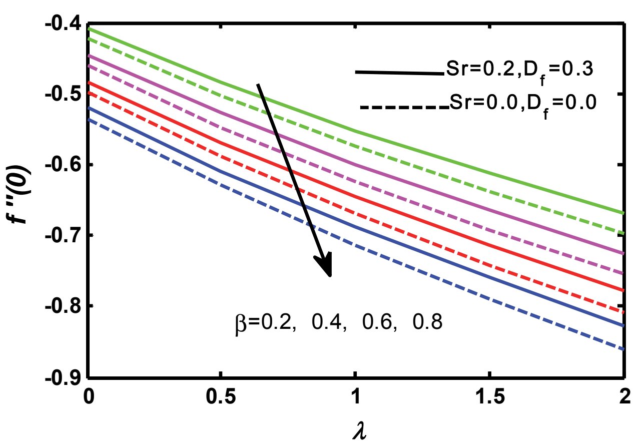


Fig.13 Effect of β on $f''(0)$ against λ

Effects of magnetic field parameter M and Prandtl number Pr on the rate of heat transfer $-\theta'(0)$ and local Sherwood number $-\phi'(0)$ for Newtonian and non-Newtonian cases are respectively shown in Figs. 14 and 15. It is noticed that both $-\theta'(0)$ and $-\phi'(0)$ decrease with the increase of M and Pr values. It is also hypothesized that the rise in β will increases the resistance of fluid motion. So, in the absence of non-Newtonian effects the present model reduces to the Newtonian model for a viscous fluid. These results clearly supported from the physical point of view. Figs. 16(a) and 16(b) respectively, display the effect of Soret number (Sr) against the ratio of relaxation to retardation time's parameter (λ) and chemical reaction parameter (Kr) on local Sherwood number $\phi'(0)$. It is observed that $\phi'(0)$ decreases as Sr increase.

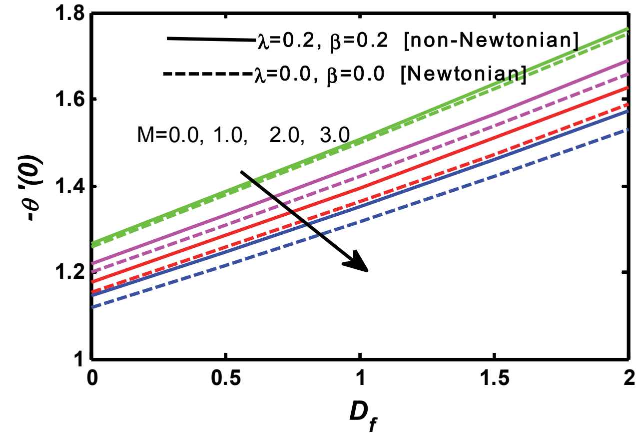


Fig.14 Effect of M on $-\theta'(0)$ against D_f

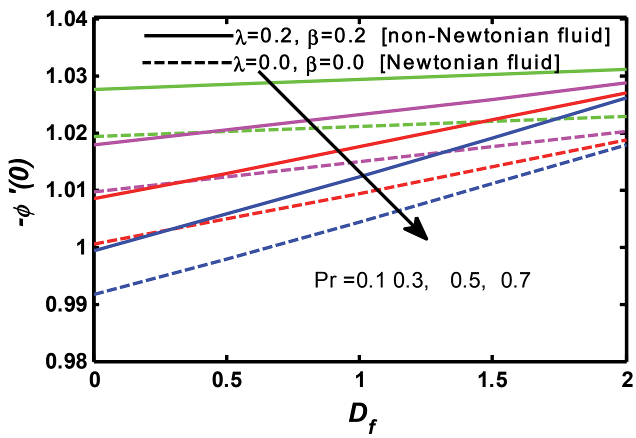


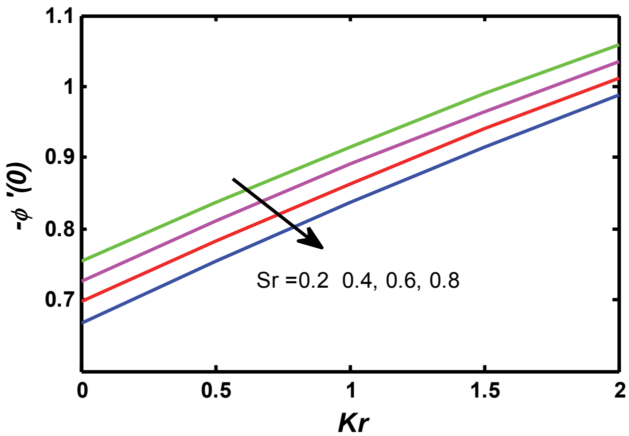
Fig. 15 Effect of Pr on $-\phi'(0)$ against D_f

5. CONCLUSIONS

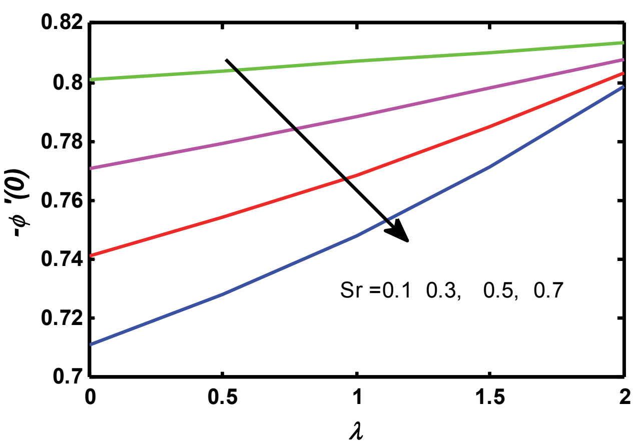
In the present investigation, we study the Soret and Dufour effects on MHD radiative heat and mass transfer flow of a Jeffrey fluid over a stretching sheet in the presence of chemical reaction. The governing equations are transformed into a system of non-linear ordinary differential equations and are then solved numerically by using fourth order Runge-Kutta- method along with shooting technique. The main observations of present research can be summarized as follows.

- The velocity profile and momentum boundary layer thickness are enhanced with the rise of Deborah number β whereas opposite trend in the temperature and concentration distributions.
- An increase in the relaxation to retardation time's parameter λ leads to a reduction in the velocity field and reverse trend in temperature and concentration profiles.
- The increase in Sr and Pr show a decrease in temperature and increase in concentration profiles.

- The fluid velocity, temperature and concentration of non-Newtonian Jeffrey fluid are less when compared to Newtonian fluid.
- The heat and mass transfer coefficients are far away from the stretching surface when the fluid changes from Newtonian to non-Newtonian (Jeffrey).



(a) β =0.2 and λ =0.2.



(b) β =0.2.

Fig. 16 Variation of $-\phi'(0)$ against a) Sr and Kr, b) Sr and λ.

NOMENCLATURE

B_0	magnetic induction [tesla]
C	concentration [kmol/m^3]
C_f	skin-friction co-efficient
c_p	specific heat at constant pressure [Jkg^{-1}K^{-1}]
C_w	species concentration at the wall [kmol/m^3]
C_∞	species concentration far from the wall [kmol/m^3]
D	diffusion coefficient [m^2/s]
F	non-dimensional stream function
f'	dimensionless velocity
K	fluid thermal conductivity [Wm^{-1}K$^{-1]}$
K_s	Roseland mean absorption coefficient
Kr^*	chemical reaction parameter
l	characteristic length
M	surface temperature parameter
m_w	rate of mass transfer
M	magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
q_r	radiative heat flux[Wm^{-2}]
q_w	rate of heat transfer
R	radiation parameter
Re_x	local Reynolds number
R_1	Rivlin-Ericksen tensor
S	extra stress tensor
Sc	Schmidt number
Sh	Sherwood number
T_w	temperature at the wall (K)
T_∞	temperature far away from the wall (K)
u, v	velocity components in the x-,y-directions respectively [ms^{-1}]
x	distance along the wall [m]
y	distance normal to the wall [m]

Greek symbols

β	Deborah number
γ	heat source/sink parameter
η	similarity variable
λ	ratio of relaxation and retardation times
λ_1	relaxation time [s]
μ	dynamic viscosity [Pa/s]
υ	kinematic viscosity [m^{2}s^{-1}]
ϕ	non-dimensional concentration
ρ	fluid density [kgm^{-3}]
σ	electric conductivity
σ^*	Stefan-Boltzman constant
θ	non-dimensional temperature
τ	Cauchy stress tensor
τ_w	skin- friction on sheet
β	Deborah number

Subscripts

w	sheet surface
∞	Infinity

Superscript

$'$	differentiation with respect to η

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