



# NATURAL CONVECTION OF NANOFUIDS PAST AN ACCELERATED VERTICAL PLATE WITH VARIABLE WALL TEMPERATURE BY PRESENCE OF THE RADIATION

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## ABSTRACT

The natural convection of the nanofluids from a vertical accelerated plate in the presence of the radiation flux and magnetic field is observed in this study. Initially, the plate with a temperature higher than the temperature of nanofluids is at rest and then it accelerates moving upward and then the wall temperature decreases. The governing unsteady equations are solved by the explicit method based on the forward finite difference. Three different types of water-based nanofluids containing copper Cu, aluminum oxide Al<sub>2</sub>O<sub>3</sub> and titanium dioxide TiO<sub>2</sub> are taken into consideration. The hydrodynamic and thermal performance of the nanofluids is calculated. The results of the computation show that the velocity profiles are influenced by the type of the nanofluids, Grashof number, radiation parameter, magnetic field and volume fraction of nanoparticles. The shear stress profiles of the nanofluid Cu-water has the lowest values. The temperature distributions are very slightly different for the same volume fraction for all nanofluids. The heat transfer from plate to the nanofluids is influenced by radiation parameter, volume fraction and Prandtl number. This study is a model of the cooling system in engineering applications.

**Keywords:** *Nanofluids, Accelerated vertical plate, Heat transfer, Natural convection*

## 1. INTRODUCTION

Fluids with nano size solid particle suspended in them have been given the name nanofluids, which in recent have shown promise as heat transfer fluid. The heat transmission performance has been a most important issue for a long period and it can be determined by new fluid that can transmit more heat than ordinary base fluid. The nanofluid meets this condition since it has high thermal performance. As it was first introduced by Choi and Eastman (1995). The fast developments in the nanoscale technology and materials are emerged as an embryonic field of science for the researchers. The enhancement of the thermal conductivity of the nanofluids has become one of the most important area of the heat mass transfer due to its wide application in engineering and industries. Attention to the heat transfer of vertical plate becomes a basic model in engineering application for design of a heat exchanger apparatus.

Chamkha and Aly (2011) investigated the effect of heat generation on convection of nanofluid past an immovable vertical plate in a steady two dimension case. In steady two dimension domain, the influence of heat flux from the flow past an immovable vertical plate on the nanofluids parameter in term of Lewis number has been studied by Bagheri *et al.* (2013) and Khan & Aziz (2011). The influence of thermal radiation and magnetic field intensity of the flow of the nanofluid past a vertical nonmoving plate with constant wall temperature is investigated by Turkyilmazoglu & Pop (2013) and Ganga *et al.* (2017). Loganathan *et al.* (2013) investigated the effect of radiation on the natural convective of nanofluid past an infinite vertical plate with constant heating temperature and constant velocity. The heat flux effect from a vertical plate to the natural convection of nanofluid

was investigated by Noghrehabadi *et al.* (2013) and Narahari *et al.* (2017). Uddin and Harmand (2013) analyzed the convection heat transfer of nanofluids along a vertical plate in a porous medium by

using the Forchheimer model. The convection of nanofluid over a constant heated vertical plate with the effect of Brownian motion and thermophoresis has been explored by Ghalambaz *et al.* (2014), Aziz *et al.* (2012) and Kuznetsov *et al.* (2014). Gnanewara (2014) gave the influence of thermal radiation on the convection of nanofluid past a vertical immovable plate with uniform heat flux. Subhashini and Sumathi (2014) give the solution of the convection of nanofluid over a vertical plate which moves in the same or opposite direction to the free stream. The influence of radiation and Soret parameter of nanofluid past a constant moving plate was explored by Raju *et al.* (2015). Rajesh *et al.* (2015) gave the study of the influence of the viscous dissipation of the nanofluid past a constant moving plate. Das and Jana (2015) give the study of the effect of the magnetic field and radiation from a constant moving plate in nanofluid flow in which initially the plate temperature equal to the ambient temperature. Similar work was carried out to investigate the effect of radiation and magnetic field on the nanofluid convection for a vertical accelerated plate (Sraavan Kumar and Rushi Kumar, 2017). The free convection of nanofluid past an oscillatory movement of the plate in the presence of a magnetic field was presented by Satya Naraya *et al.* (2015) and Sheri & Thumma (2018). For oscillatory temperature effect on free convection of nanofluid past a vertical plate was investigated by Ranga and Rao (2016). The magnetohydrodynamic effect on nanofluid past an accelerated vertical plate with constant temperature was studied by Geetha *et al.* (2017). The influence of heat generation from two vertical plates on the hydromagnetic convective flow of nanofluid was presented by Prakash and Suriyakumar (2017). For the study of the effect of the heat flux and heat source on free convection of nanofluid over an accelerated plate was analyzed by Azhar *et al.* (2017). The analysis of the effect of Hall current on the convection in radiative magneto nanofluid is shown by Sharma *et al.* (2017). The effects of Diffusion and thermal radiation absorption on MHD free convective heat and mass transfer flow of a nanofluid bounded by a semi-infinite flat plate moving with constant velocity were analyzed by Prasad *et al.* (2018). In case of the nanoparticle shape effect on the free convection of flow past a vertical plate in a steady regime was

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given by Sobamowo (2019). The free convection flow of nanofluid past an infinite accelerated plate with increasing wall temperature and damped thermal flux was shown by Nisa *et al.* (2019). Vemula *et al.* (2016) give the study of the effect of the unsteady free convection of nanofluid past the accelerated vertical plate with uniform wall temperature and radiation. They showed the effect of the increase of temperature and the exponential acceleration of the vertical plate on the convection of nanofluid.

Based on the above studies we have considered the unsteady natural convection and heat transfer flow of nanofluids past an infinite vertical plate with radiation and magnetic field. We have extended the work of Kaprawi (2015) and with the novelty of considering magnetic field and radiation in nanofluids. In this study, the plate is initially at higher temperature than the fluid, so there is a heat transfer from plate to the nanofluid and due to the heat transfer thus the wall plate temperature decreases with time. The example engineering application of the cooling system is found in the cooling of hot urea granular falling from a prilling tower by gravitation acceleration, in this case the surface of the urea may behaves as the vertical plate. Other application is in the heat treatment process of a metal. A metal, after it is heated at certain temperature and then submerged in the cooling fluid.

## 2. FORMULATION OF THE PROBLEM

Consider the unsteady natural convective flow and heat transfer of a nanofluid past a vertical accelerated vertical plate with non-uniform wall temperature, in this case, the wall temperature decreases with time. A magnetic field of uniform strength is applied perpendicular to the plate and a radiative heat flux  $q_r$  is applied in the normal direction to the plate. At time  $t \leq 0$ , the plate is maintained at the temperature higher than ambient temperature  $T_\infty$ . In this condition, both the plate and the nanofluid are at rest. At time  $t > 0$ , the plate accelerates with increasing time in its own plane in the  $x$ -direction with velocity  $u_p = U_0 (1 - e^{-at})$  and the wall temperature decreases with time or  $T_w = 1/(1+at)$ . It is assumed that the viscous dissipation is neglected. A uniform transverse magnetic field of strength  $B_0$  is applied parallel to the  $y$ -axis. The fluid is a water-based nanofluid containing three type nanoparticles Cu,  $Al_2O_3$  and  $TiO_2$  whose thermal properties are given in Table 1 and the nanoparticles in a spherical shape. It is further assumed that the base fluid and the suspended nanofluid particles are in thermal equilibrium.

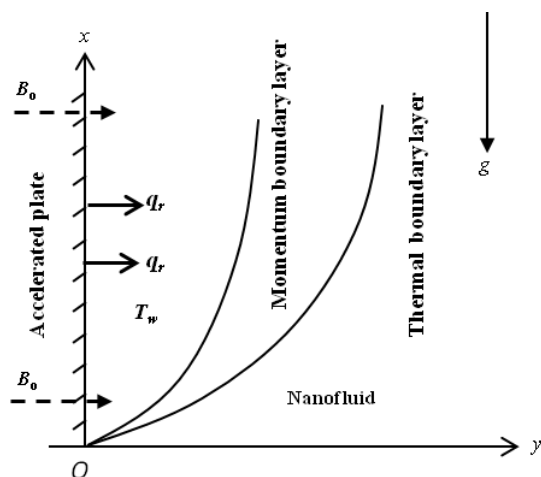


Fig. 1 Geometry of the problem

By adopting the Boussinesq approximation, the governing equation corresponding to the nanofluid model are given as follows (Das *et al.*,2015) :

$$\rho_{nf} \frac{\partial u}{\partial t} = \mu_{nf} \frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf}(T - T_\infty) - \sigma_{nf} B_0^2 u \quad (1)$$

$$(\rho c_p)_{nf} \frac{\partial T}{\partial t} = k_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

where  $\rho_{nf}$  is the density of the nanofluid,  $u$  the velocity along the  $x$ -direction,  $\mu_{nf}$  the dynamic viscosity of the nanofluid,  $\beta_{nf}$  the thermal expansion coefficient of the nanofluid,  $g$  the acceleration due gravitation,  $T$  the temperature,  $\sigma_{nf}$  the electrical conductivity of the nanofluid,  $k_{nf}$  the thermal conductivity of the nanofluid,  $q_r$  the radiative heatflux and  $c_p$  the specific heat at constant pressure. The initial and boundary conditions are

$$t = 0 : u = 0, \quad T = T_w \quad \text{for } y = 0$$

$$T = T_\infty \quad \text{for all } y > 0$$

$$t > 0 : u = u_0(1 - e^{-at}), \quad T = T_\infty + (T_w - T_\infty)/(1+at) \quad \text{at } y = 0,$$

$$u = 0, \quad T = T_\infty \quad \text{at } y \rightarrow \infty$$

The other parameters of nanofluid in the eq.( 2) are defined by

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s,$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}},$$

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s, \quad (3)$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s,$$

$$\sigma_{nf} = \sigma \left[ 1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} \right], \quad \sigma = \frac{\sigma_s}{\sigma_f}$$

where  $\phi$  is the solid volume fraction of the nanoparticle,  $\rho_f$  the density of the base fluid,  $\rho_s$  the density of the nanoparticle,  $\sigma_s$  the electrical conductivity of the nanoparticle,  $\sigma_f$  and the electrical conductivity of the base fluid respectively,  $(\rho c_p)_f$  the heat capacitance of the base fluid,  $(\rho c_p)_s$  the heat capacitance of the nanoparticle. The effective thermal conductivity of the nanofluid given by Hamilton and Crosser model from Kakac and Pramuanjaroenkij (2009) is given by

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (4)$$

where  $k_f$  is the thermal conductivity of the base fluid and  $k_s$  the thermal conductivity of the nanoparticles.

Table 1 Thermo physical properties of water and nanoparticles (Oztop, 2008 and Das *et al.*,2015 )

Physical properties	Water/base fluid	Cu (Copper)	$Al_2O_3$ (Alumina)	$TiO_2$ (Titanium dioxide)
$\rho$ (kg/m <sup>3</sup> )	997.1	8933	3970	4250
$c_p$ (J/kg K)	4179	385	765	686.2
$k$ (W/mK)	0.613	401	40	8.9538
$\beta \times 10^5$ (K <sup>-1</sup> )	21	1.67	0.85	0.90
$\phi$	0	0.05	0.15	0.2
$\sigma$ (S/m)	$5.5 \times 10^{-6}$	$59.6 \times 10^6$	$35 \times 10^6$	$2.6 \times 10^6$

The Rosseland approximation of the radiative heat flux term (Hakeem and Sathiyathan,2009) is given by

$$q_r = -\frac{4\sigma^+}{3k^+} \frac{\partial T^4}{\partial y} \quad (5)$$

where  $\sigma^+ = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$  is the Stefan-Boltzmann constant and  $k^+$  the Rosseland mean absorption coefficient. It is assumed that the temperature difference within the flow is small enough such that the term  $T^4$  may be expressed as a linear function of temperature. It is done by expanding  $T^4$  in a Taylor series about a free stream temperature  $T_\infty$  as follows

$$T^4 = T_\infty^4 + 3T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \dots \quad (6)$$

Neglecting higher order term in eq. (6), so we have

$$T^4 \approx 4T_\infty T - 3T_\infty^3 \quad (7)$$

By substituting eq.(7) into eq. (5) and eq. (2), it gives

$$\frac{\partial T}{\partial t} = \left( \frac{k_{nf}}{(\rho c_p)_{nf}} + \frac{16\sigma^+ T_\infty^4}{3k^+(\rho c_p)_{nf}} \right) \frac{\partial^2 T}{\partial y^2} \quad (8)$$

The dimensionless variables as follows

$$U = \frac{u}{u_o}, \quad \tau = \frac{tu_0^2}{\nu_f}, \quad Y = \frac{yu_o}{\nu_f}, \quad \theta = \frac{T - T_\infty}{T_s - T_\infty}, \quad A = \frac{a\nu_f}{u_o^2},$$

$$Gr = \frac{g\beta_f \nu_f (T_s - T_\infty)}{u_o^2}, \quad Pr = \frac{\mu_f c_p}{k_f}, \quad M = \frac{\sigma_f B_o^2 \nu_f}{\rho_f u_o^2} \quad (9)$$

$$Nr = \frac{16\sigma^+ T_\infty^3}{3k_f k^+}, \quad \alpha = \frac{S}{\rho_f u_o^2}, \quad L = \frac{lu_0}{\nu_f}$$

where  $Gr$  is the Grashof number,  $Pr$  the Prandtl number,  $M$  the magnetic parameter,  $Nr$  the radiation parameter,  $\alpha$  the dimensionless shear stress.

After introducing the eq.(9) into the eq.(1) and (8), these equations become

$$\frac{\partial U}{\partial \tau} = b_1 \frac{\partial^2 U}{\partial Y^2} + Grb_2 \theta - Mb_3 U \quad (10)$$

$$\frac{\partial \theta}{\partial \tau} = b_4 \frac{\partial^2 \theta}{\partial Y^2} \quad (11)$$

Initial and boundary condition become

$$t = 0 : U = 0, \quad \theta = 1 \quad \text{for } Y = 0$$

$$\theta = 0 \quad \text{for all } Y > 0$$

$$t > 0 : U = (1 - e^{-A\tau}), \quad \theta = \frac{1}{1 + A\tau} \quad \text{at } Y = 0, \quad (12)$$

$$U = 0, \quad \theta = 0 \quad \text{at } Y \rightarrow \infty$$

The coefficients of the eq.(10) and (11) as follows

$$b_1 = \frac{1}{(1-\phi)^{2.5} \left[ (1-\phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right]}, \quad b_2 = \frac{\left[ (1-\phi) + \phi \left( \frac{\rho\beta \right)_s}{(\rho\beta)_f} \right]}{\left[ (1-\phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right]}$$

$$b_3 = \frac{\left[ 1 + \frac{3(\sigma-1)\phi}{(\sigma+2) - (\sigma-1)\phi} \right]}{\left[ (1-\phi) + \phi \left( \frac{\rho_s}{\rho_f} \right) \right]}, \quad b_4 = \frac{\left[ \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right] + Nr}{Pr \left[ (1-\phi) + \phi \left( \frac{\rho c_p}{\rho_f} \right)_s \right]}$$

### 3. METHOD OF SOLUTION

The set of coupled linear partial differential equations (10) - (11) of the parabolic type and are to be solved by using the initial and boundary conditions of the equation (12). We use the forward finite difference and these equations become

$$\frac{U_{i,j+1} - U_{i,j}}{\Delta \tau} = b_1 \left( \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta Y^2} \right) + b_2 Gr \theta_{i,j} - b_3 M U_{i,j} \quad (13)$$

$$\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta \tau} = b_4 \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta Y^2} \right) \quad (14)$$

The suffix  $i$  refer to the  $Y$ -direction and  $j$  to time. The explicit method is used to solve the above equations (13) and (14). The explicit method is stable when the time increment is small enough and therefore  $\Delta \tau = 0.001$  while  $\Delta Y = 0.05$  in  $Y$ -direction. All of  $U$  and  $\theta$  with suffix  $j$  are dropped into the right part of the equations, so the values of  $U_{i,j+1}$  are obtained from  $U_{i,j}$  and  $\theta_{i,j+1}$  from  $\theta_{i,j}$ . For constant point of  $j$ , we calculated  $U_{i,j+1}$ ,  $\theta_{i,j+1}$  with  $i = 0, 1, 2, 3, \dots, m$ . The point  $j$  was increased and  $U_{i,j+1}$ ,  $\theta_{i,j+1}$  with  $i = 0, 1, 2, 3, \dots, m$  were calculated. The same procedure was done until  $j = n$ .

After the velocity and the temperature are obtained so the dimensionless shear stress  $\alpha$  and Nusselt number  $Nu$  are calculated respectively as follows

$$\alpha = \frac{1}{(1-\phi)^{2.5}} \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \quad (15)$$

The Nusselt number and heat transfer are defined by

$$Nu = \frac{hl}{k_f} \quad (16)$$

$$q_r = h(T_s - T_\infty) = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad (17)$$

By substituting eq.(17) into eq. (16) and using dimensionless variables of eq. (9), the Nusselt number is written as

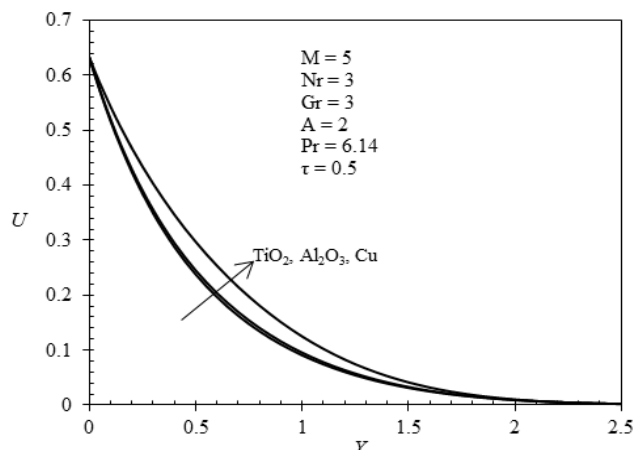
$$\frac{Nu}{L} = -\frac{k_{nf}}{k_f} \left( \frac{\partial \theta}{\partial Y} \right)_{Y=0} \quad (18)$$

### 4. RESULTS AND DISCUSSION

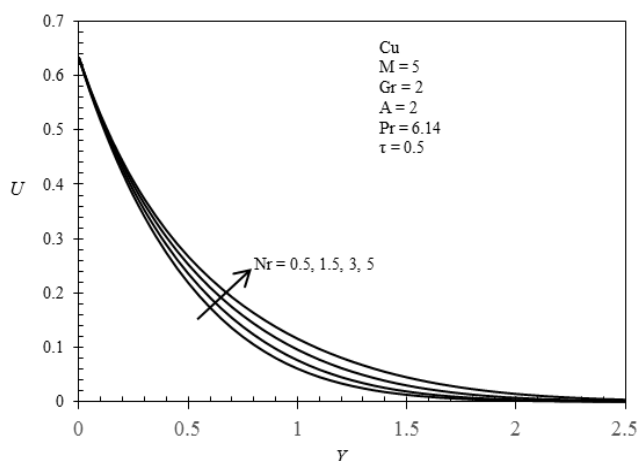
The numerical solutions were achieved by a computer using the software of Just Basic version v2.0. The calculations were carried out for different values of  $\phi$  as stated in Table 1 and if not, it is stated in the text. We have presented the dimensionless velocity  $U$ , dimensionless temperature  $\theta$ , skin friction and Nusselt number in Figs.1-10 for several values of  $M$ ,  $Nr$ ,  $Gr$ , and  $\tau$ . Fig. 2 shows the velocity profiles of different nanoparticles. It is observed that the velocity profiles are the same for  $TiO_2$ -water and  $Al_2O_3$ -water as their densities and thermal expansion coefficients are closed to each other's value, but it is slightly difference for  $Cu$  nanoparticle in the region of  $0 < Y < 1.7$  and outside of this region  $Y > 1.7$ , the curves meet together which approach to zero velocity, so the hydrodynamic boundary layers have the same thickness. For  $Y = 0$  the velocities have the same values since there is no slip condition between the nanofluid and the surface and it is due to the boundary condition at the wall.

Fig. 3 shows the velocity of the base fluid with nanoparticle  $Cu$  for different values of the radiation parameter  $Nr$  and at time  $\tau = 0.5$ . The velocities increase with the increase of  $Nr$  and the profiles have a similar pattern, but when farther away from the surface, the difference of the velocity values are observed. The velocities approach zero at  $Y \approx 1.8$  for  $Nr = 0.5$  and  $Y \approx 2.4$  for  $Nr = 5$  which show that the momentum boundary layer thickness increases with the increase of  $Nr$ . Fig.4 depicts the velocity profiles for several Grashof number  $Gr$  and for  $Cu$ -water nanofluid. We observe that  $Gr$  effects significantly the velocity distributions and the velocity descend rapidly for small values of  $Gr$ . All curves move to zero at  $1.8 < Y < 2.4$  and it seems that the boundary layer thickness has the same as in the effect of  $Nr$ . The significant effect of  $Gr$  on the velocity variations is due to the direct effect of  $Gr$  parameter in the momentum equation. The velocity profile of nanofluid  $Cu$ -water for several values of time is shown in Fig.5. It is observed that the velocity at the wall at  $Y = 0$  is the boundary condition that rises exponentially with time. The curves have similar pattern each other and the profiles tend to coincide at higher  $\tau$ . The effect of the magnetic field is shown by Fig.6 for  $Al_2O_3$ -water. As seen an increase in magnetic field  $M$  along the wall causes to decrease the velocity of nanofluid. The boundary layer seems to be the same about 2.5 for all  $M$  that shows that the variation of  $M$  does not depend on the boundary layer thickness. The velocity variations are significant in the middle between the wall and  $Y \approx 2.0$ . The effect magnetic field on an electrically, conducting fluid

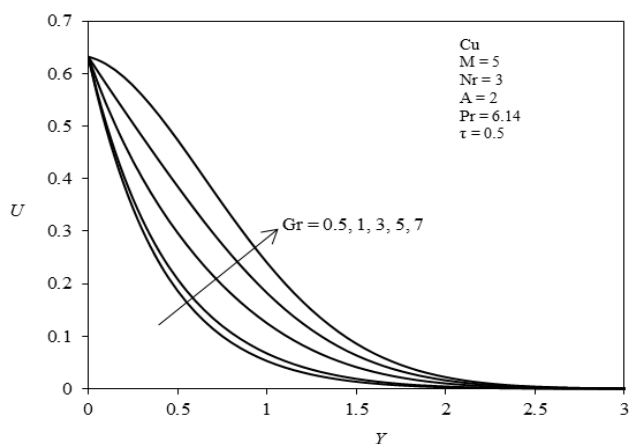
gives rise to a resistive-type force called Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer.



**Fig. 2** Velocity profiles with different nanoparticles

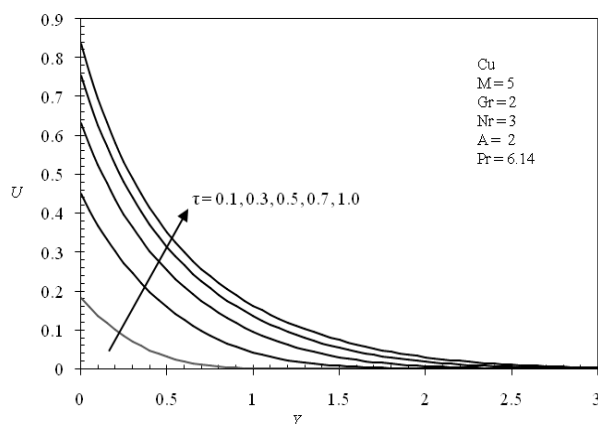


**Fig. 3** Velocity profiles for difference value of Nr

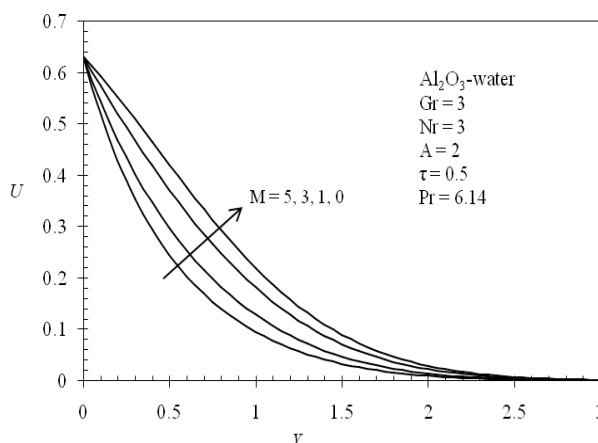


**Fig. 4** Velocity profiles with different Gr

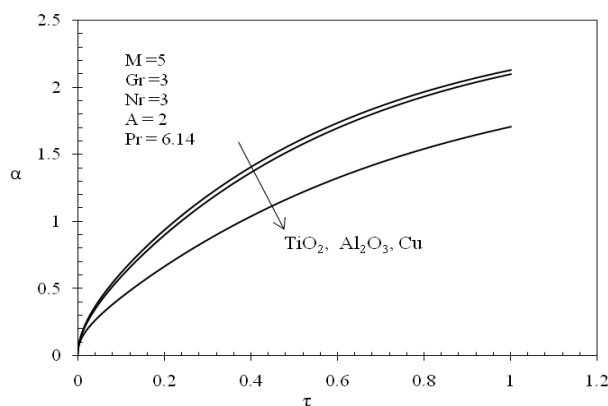
The shear stress of the nanofluid  $\text{TiO}_2$ -water,  $\text{Al}_2\text{O}_3$ -water and Cu-water are represented by Fig. 7. As seen from the curves show that the nanofluid of Cu-water has the lowest shear stress. As it is noted that the shear stress represents the velocity of the wall since no-slip condition between the wall and the nanofluid and the profiles is represented by the boundary condition at the wall  $U = 1 - e^{-A\tau}$ . For  $\text{TiO}_2$  and  $\text{Al}_2\text{O}_3$ -water, the shear stress is almost the same each other since the thermo-physical properties of both particles have no far difference as stated in Table 1.



**Fig. 5** Velocity profiles with different  $\tau$



**Fig. 6** Velocity with different Magnetic field



**Fig. 7** Shear stress for different nanofluids

For volume fraction of the three types of nanofluids is constant  $\phi = 0.1$ , the shear stress is represented in Fig. 8. It can be seen that Cu-water has still lowest shear stress compared to  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$ . The magnetic field affects significantly the shear stress profiles as shown by Fig. 9. It can be seen that the shear stresses are the same for small  $\tau$ . The shear stress increase with time for  $M > 1$ , but below this value when  $\tau$  increases, the shear stress seems to be constant for  $M = 1$  and even decreases when the magnet field is not present  $M = 0$ . It is noted that the shear stress represents the slope of the velocity profiles at the wall in which the bigger the slope, the greater the shear stress.

The example of the temperature distributions for different radiation parameter Nr and for nanofluid  $\text{TiO}_2$ -water is shown by Fig. 10. The fluid temperature increases with the increase of the radiation parameter. The increase in radiation parameter means the increase of energy absorbed by the fluid and so the temperature increases. the

increase in temperature results in a rising thermal boundary layer thickness. The comparison of the different nanofluid Cu-water,  $Al_2O_3$ -water and  $TiO_2$ -water, velocity profiles at  $\tau = 0.5$  is shown by Fig. 11. The slight difference in velocity values are observed for three types of the nanofluids, but the boundary layer thickness is the same for all nanofluids. The temperature of Cu-water nanofluids is found to be lower than  $Al_2O_3$ -water and  $TiO_2$ -water nanofluids since the volume fraction is lower  $\phi = 0.05$  for Cu-water,  $\phi = 0.15$  for  $Al_2O_3$ -water and  $\phi = 0.2$  for  $TiO_2$ -water. Although nanoparticle Cu has high thermal conductivity compared to  $Al_2O_3$  and  $TiO_2$  but the temperatures are still lower. For the values of all nanofluids with volume fraction  $\phi = 0.1$  the temperature distributions are practically the same as shown by Fig. 12, the differences are very small. It is noted that at  $Y = 0$  is the boundary condition at the wall where the nanofluid temperatures equal to the wall temperature. The effect of the volume fraction parameter of Cu seems to significant in increasing the temperature distribution since more heat absorbed by particle Cu in the nanofluid of Cu-water.

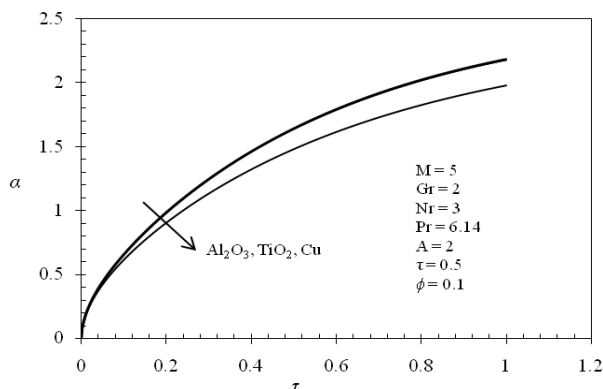


Fig. 8 Shear stress of nanofluids for constant  $\phi$

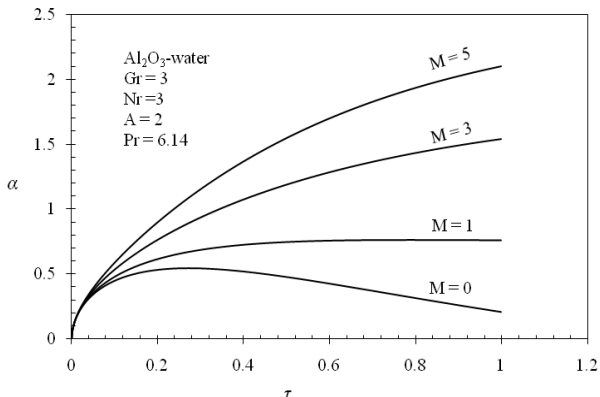


Fig. 9 Shear stress of  $Al_2O_3$ -water for different M

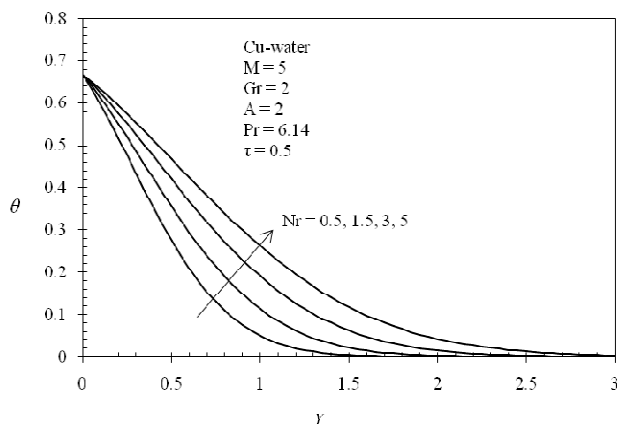


Fig. 10 Temperature profiles for different Radiation parameter Nr

Figure 13 shows the effect of time  $\tau$  on the temperature distributions for nanofluid  $TiO_2$ -water. It reveals that the temperature at  $\tau = 0.1$  decreases rapidly to zero at about  $Y = 1.2$  and the zero velocities shift to large values of  $Y$  when  $\tau$  increase and therefore, the thermal boundary layer increases. At point  $Y = 0$  the temperature of nanofluid decreases with time which is due to the boundary condition at the wall. In this study, the dimensionless time is limited to  $\tau \leq 1$ . It can be predicted that when  $\tau$  increase larger than unity, the temperature profiles become slender.

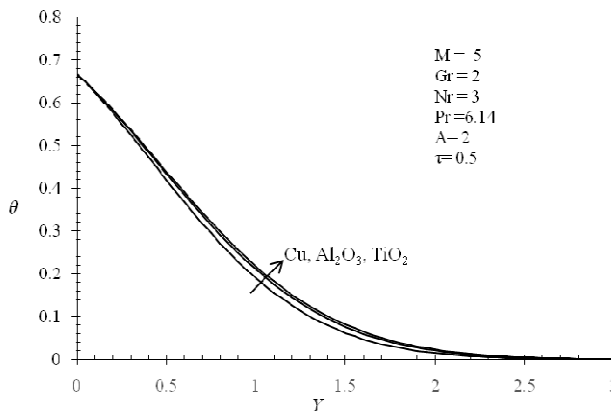


Fig. 11 Temperature profiles for different nanofluids and  $\phi$

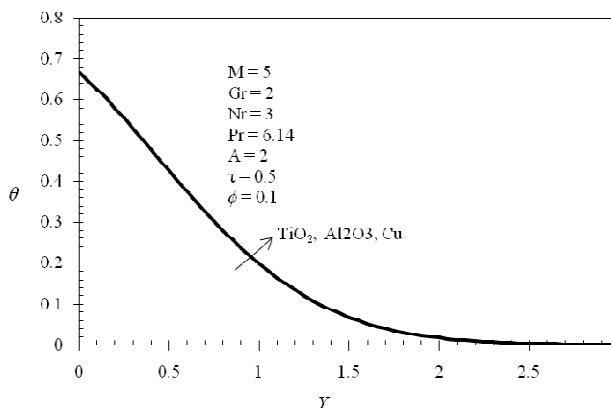


Fig. 12 Temperature profiles with different nanofluid and  $\phi = 0.1$

The heat transfer that is represented by Nusselt number and this number shows the heat released by the accelerated plate to the nanofluid. High Nusselt numbers indicate high heat transfer between the two media and this number represents the temperature gradient at the wall of the plate. Fig. 14-16 depicts the effect of the radiation parameter Nr on Nusselt number for nanofluid of  $Al_2O_3$ -water, the effect of volume fraction and the Prandtl number. All curves show similar pattern profiles. For other nanofluids of Cu and  $TiO_2$ , the Nusselt number is not very much different since the temperature are similar (Fig. 12). The curves show that the increase in the radiation parameter Nr causes the number of Nu to decrease. The Nusselt number decreases with the increase of time  $\tau$ . The plate initially  $\tau = 0$  at high temperature, the Nusselt is high and then the heat transfer decreases rapidly in the range of  $\tau < 0.2$ . For time higher the value, the Nusselt number decreases slowly with time since the plate accelerates slowly and has less heat to be released to the nanofluid. As it noted that in forced convective heat transfer, the Nusselt number depends on the Reynolds number that means the velocity. Hence, the heat transfer in the presence of radiation, it is a better heat transfer process for low radiation intensity. The volume fraction has a small effect on the Nusselt number profiles as shown by Fig. 15. The Increase in  $\phi$  will increase in Nu. The effect of the Prandtl number on the Nusselt number as represented by Fig. 16. A high Pr indicates high heat transfer.

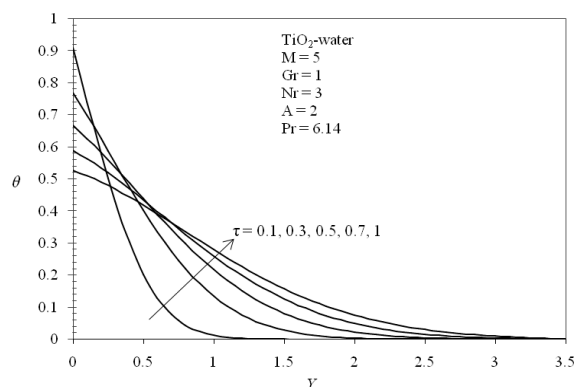


Fig. 13 Temperature profiles for different  $\tau$

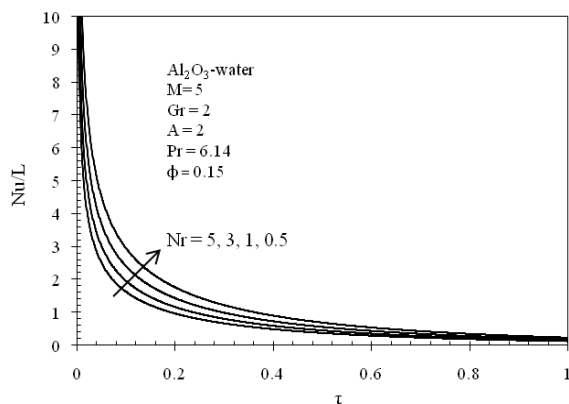


Fig. 14 Nusselt number for different  $Nr$

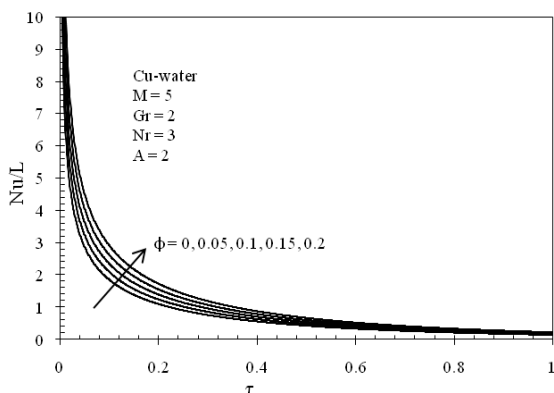


Fig. 15 Nusselt number for different volume fraction

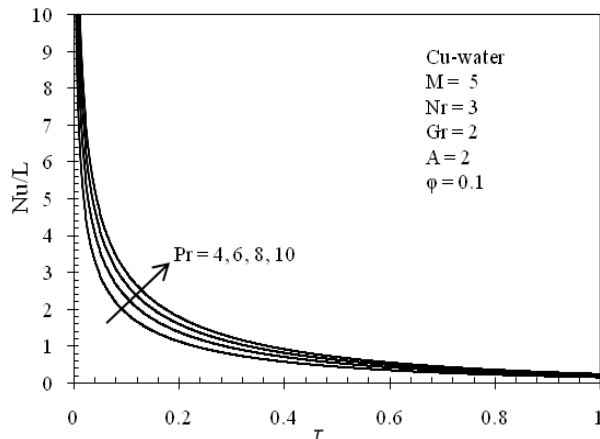


Fig. 16 Nusselt number for different  $Pr$

### 3. CONCLUSIONS

The numerical study of the free convection of the nanofluids Cu-water,  $Al_2O_3$ -water and  $TiO_2$ -water by the presence of the radiation parameter and magnetic field from the accelerated vertical plate has been presented. The plate temperature descend with time, and hence the heat transfer occurs from the plate to the nanofluids. The radiation parameter and the magnetic field have the effect on the temperature, velocity, shear stress and heat transfer rate. The velocity of nanofluids rises with rising radiation parameter and the velocity decreases with the increase of the magnetic field. The shear stress of the nanofluid Cu-water has the lowest value. The heat transfer is high for low radiation parameter value, for high volume fraction and for high Prandtl number.

This study provides information and limitations of the use of the nanofluids in designing a cooling system in the engineering field. The thermal performance of the three types of the nanofluids are not much different and it may be important to observe other nanofluids which may have higher heat transfer rate.

### NOMENCLATURE

$a$	time constant ( $s^{-1}$ )
$A$	dimensionless time constant
$c_p$	specific heat ( $J/kg \cdot K$ )
$g$	gravitational acceleration by gravitation ( $m/s^2$ )
$k$	thermal conductivity ( $W/m \cdot K$ )
$l$	characteristic length (m)
$L$	dimensionless characteristic length
$M$	magnetic parameter
$q_r$	radiation heat flux ( $W/m^2$ )
$t$	time (s)
$T$	temperature (K)
$u$	velocity (m/s)
$U$	dimensionless velocity
$x, y$	coordinates (m)
$Y$	dimensionless coordinate
$Gr$	Grashof number
$Nu$	Nusselt number
$Pr$	Prandtl number
$Nr$	radiation parameter

### Greek Symbols

$\beta$	thermal expansion coefficient ( $K^{-1}$ )
$\nu$	kinematic viscosity ( $m^2/s$ )
$\mu$	dynamic viscosity ( $NS/m^2$ )
$\phi$	volume fraction
$\theta$	dimensionless temperature
$\rho$	density ( $kg/m^3$ )
$\sigma$	electrical conductivity ( $S/m$ )
$\tau$	dimensionless time

### Subscripts

$nf$	nanofluids
$s$	nanoparticle
$f$	fluid
$p$	plate
$\infty$	ambient environment

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