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MELTING AND RADIATION EFFECTS ON MIXED CONVECTION BOUNDARY LAYER VISCOUS FLOW OVER A VERTICAL PLATE IN PRESENCE OF HOMOGENEOUS HIGHER ORDER CHEMICAL REACTION

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ABSTRACT

The present paper investigates the combined effects of melting phenomenon and viscous dissipation over a steady incompressible mixed convection boundary layer fluid flow along a vertical plate. Radiation and double dispersion are also taken into consideration. Further effect of homogeneous chemical reaction of order 'n' is studied over the non-Darcy porous plate. Continuum equations that characterize fluid flow are transformed to a set of non linear ordinary differential equations through a suitable similarity transformation. These equations are then solved by MATLAB 'bvp4c' iterative programming method. As a matter of accuracy and validation, available results are compared with the present study as a special case. Flow characteristics of the problem are illustrated graphically.

Keywords: *Chemical Reaction, Heat and Mass Transfer, Porous Medium, Melting Effect, Mixed Convection, Viscous Dissipation*

1. INTRODUCTION

Heat and mass transfer over viscous geometries embedded in porous media has a considerable interest of many engineering and geophysical applications such as geothermal reservoirs, process of drying porous solids, Magnetohydrodynamics(MHD) power generators and underground energy transport. Considerable literature can be found over combined heat and mass transfer in free convection boundary layer flow over heated surfaces along different geometries. Few researchers (e.g., Gebhart *et al.*, 1989; Kandasamy *et al.*, 2006; Chaudhary *et al.*, 2006), investigated the impact of radiation on heat transfer in MHD mixed convection flow with viscous dissipation and ohmic heating effects. A problem of analytical conjugate phenomena of heat and mass transfer was examined by Noor *et al.* (2012). A discussion on the influence of free convective effects on Stokes problem when the fluid is driven by applied magnetic field was focused by Soundalgekar *et al.* (1979). Effect of align magnetic field in a forced convection boundary layer flow was examined by Seddeek (2002). A study was made over mixed convection flow through permeable vertical surface in presence of radiation by Aydin and Kaya (2008).

The unique phenomena used in magma solidification, preparation of semi conductor materials and the systems where solid-liquid change materials is melting phenomena. Firstly, in absence of porous medium, Roberts (1958) examined the steady state effects when ice was placed in a hot stream of air. Later Epstein and Cho (1976) observed heat transport in a submerged bodies undergoing melting. Few authors explored the aspects of the flow with melting phenomena (e.g., Gorla *et al.*, 1999; Cheng and Lin, 2007). These studies revealed that melting process reduces the heat transfer through the solid–liquid interface. Effect of melting on mixed convective flow over a permeable vertical surface was feigned by Ahmad and Pop (2014). They found dual solution for some specific values of mixed convection parameter. The results also indicate that the melting phenomena reduces the heat transfer rate and expedites the boundary layer separation. Further Sobha *et al.* (2010) identified the nature of velocity across melting parameter in both aiding and opposing flows. Also in this study, it is observed that Nusselt number decreases with melting parameter and increases with increase in thermal dispersion. Prasad and Hemalatha (2010) examined the combined effects of radiation and melting over a vertical wall. Radiation effect on heat transfer in a flow over a vertical surface with uniform surface temperature was noticed by Hossain and Takhar (1996). Thermal dispersion and radiation effects on non-Darcy free convection flow through vertical plate were explained by Abbas *et al.* (2008). They observed that increasing radiation parameter enhances the momentum boundary layer thickness. Further Nusselt number increases with radiation parameter. Melting effect on a convective heat transfer between a melting body and surrounding fluid was discovered in Tien and Yen (1965). They further noticed that melting retards the rate of heat transfer.

It is evident from the literature that no work has been carried out on combined effects of viscous dissipation and melting phenomena in a

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mixed convection boundary layer chemically reacted fluid flow in presence of radiation over a permeable surface. So, in this paper, these effects on heat and mass transfer and as well as fluid velocity, temperature and concentration with in the boundary layer are investigated. The results are compared with Hemalatha *et al.* (2015) as a special case and found in good agreement.

2. MATHEMATICAL ANALYSIS

A problem of mixed convection steady state boundary layer flow through a vertical surface with melting phenomena is considered. Assume that the surface forms an interface between solid and liquid phases at the time of melting inside porous matrix. Plate is kept at constant temperature T_p. Further the temperatures T_s and T_∞ of solid phase and liquid phase far from the interface and plate respectively are assumed to be constant. Taking viscous dissipation along with radiation, the boundary layer equations are framed as

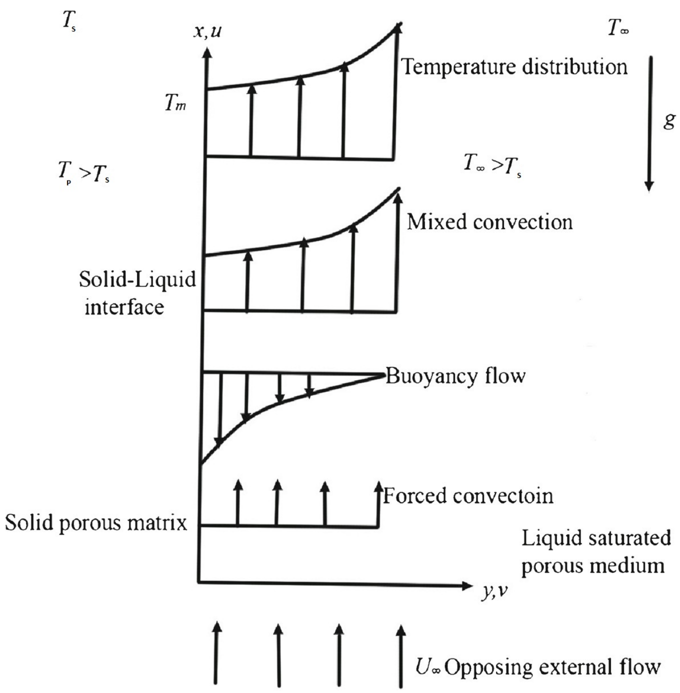

Fig. 1 Physical Model of the Problem (Hemalatha *et al.*, 2015)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} + \frac{c_f\sqrt{k}}{\nu}\frac{\partial}{\partial y}(u^2) = \frac{\rho g k}{\nu}\left(\beta_T\frac{\partial T}{\partial y} + \beta_C\frac{\partial C}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_\tau\frac{\partial T}{\partial y}\right) - \frac{1}{\rho c_p}\frac{\partial Q_r}{\partial y} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D_s\frac{\partial C}{\partial y}\right) - K_n\left(C - C_\infty\right)^n \tag{4}$$

where u, v are velocity components along x, y directions respectively, c_f is the Forchheimer constant, ν is kinematic viscosity of the fluid, g is acceleration due to gravity, k is permeability of the porous medium, c_p is specific heat at constant pressure, β_T is coefficient of thermal expansion, β_C is coefficient of solute expansion, $D_s$is thermal solute diffusivity, K_n is chemical reaction rate of order n, ρ is fluid density and C_∞ is free stream concentration, μ is dynamic viscosity of the fluid. The boundary conditions which govern the fluid flow are

$$k_{et}\frac{\partial T}{\partial y} = \rho\left[L + c_s(T_p - T_s)\right]v, T = T_p, C = C_w, \text{at} \quad y = 0 \text{ and}$$
$$u \to u_\infty, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \tag{5}$$

where L, c_s are latent heat of solid and specific heat of solid phase respectively. $C_w$is concentration of fluid near the wall and $u_\infty$is free stream velocity. In presence of mechanical dispersion one can write thermal diffusivity $\alpha_\tau = \alpha_m + \alpha_d$ where α_m is molecular diffusivity and $\alpha_d = Dud$, is dispersion thermal diffusivity. Further solutal diffusivity $D_s = D_m + D_d$ where D_m,is molecular solutal diffusivity and $D_d = \beta ud$, is dispersion solutal diffusivity. D and β are proportionality constants and d, is particle mean diameter. The effective thermal conductivity of the porous medium $k_{et} = \alpha_\tau\rho c_p$. Assume further the temperature gradient is sufficiently small within the flow, allows us to represent T^4 as a linear function of temperature. Then the variation in radiative heat flux based on Rosseland's approximation may be modeled as

$$\frac{\partial Q_r}{\partial y} = -\frac{16\widetilde{\sigma}T_p^3}{3k^*}\frac{\partial T}{\partial y} \tag{6}$$

where $\widetilde{\sigma}$, is the Stefan-Boltzmanconstant, k^*, is mean absorption coefficient. Define a stream function $\psi = f(\eta)\sqrt{\alpha_m u_\infty x}$ which satisfies $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. Velocity components in non dimension form may be derived as

$$u = u_\infty f'(\eta), \quad v = -\frac{1}{2}\sqrt{\frac{\alpha_m u_\infty}{x}}[f(\eta) - \eta f'(\eta)] \tag{7}$$

where η is the similarity variable defined by

$$\eta(x, y) = \frac{y}{x}\sqrt{\frac{u_\infty x}{\alpha_m}} \tag{8}$$

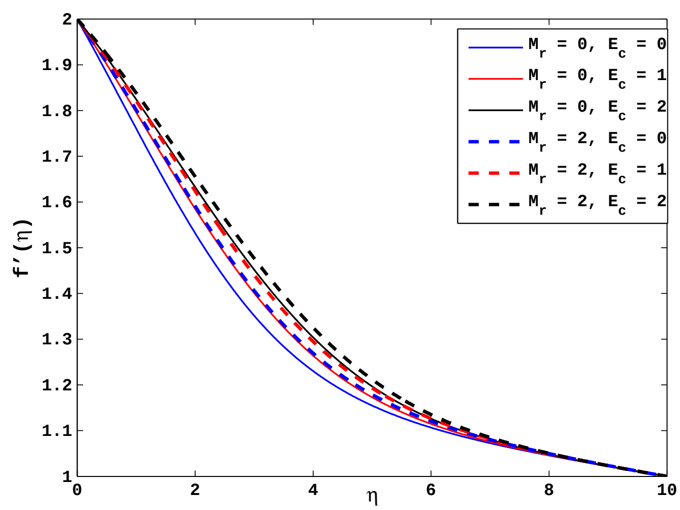

Fig. 2 Melting and viscous dissipation effects on velocity profile for $Ra/Pe = F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = n = \Gamma = 1$

Further introduce another set of non-dimensional quantities, temperature and concentration, as

$$\theta(\eta) = \frac{T - T_p}{T_\infty - T_p}, \quad \phi(\eta) = \frac{C - C_w}{C_\infty - C_w} \tag{9}$$

The equations (6)–(9) transform equations (2)–(5) in the following undimensional form

$$f''(1 + F_r f') = -\frac{Ra}{Pe}(\theta' + N_b\phi') \tag{10}$$

$$\theta''(1 + \gamma f' + \frac{4}{3}N_r) = -\frac{1}{2}f\theta' - \gamma f''\theta' - PrEcf''^2 \tag{11}$$

$$\phi''(\frac{1}{Le} + \gamma^* f') = -\gamma^*\phi' f'' - \frac{1}{2}f\phi' + \Gamma\phi^n \tag{12}$$

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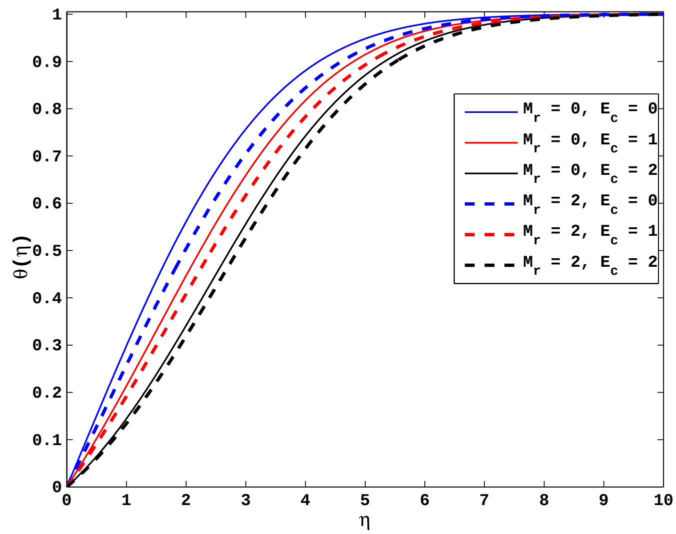

Fig. 3 Melting and viscous dissipation effects on temperature profile for $Ra/Pe = F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = n = \Gamma = 1$

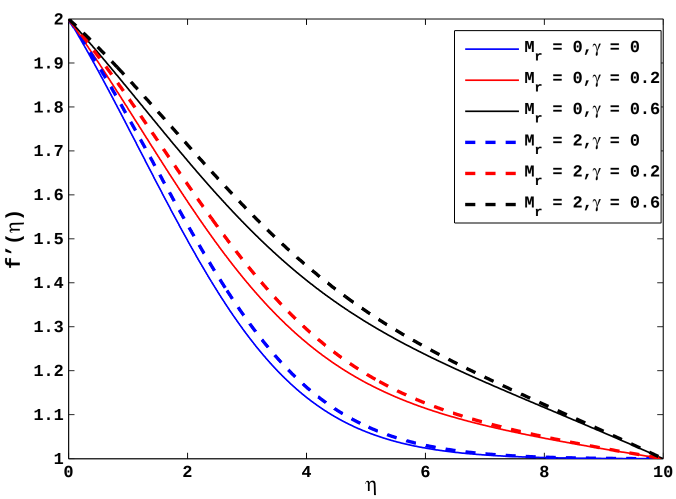

Fig. 4 Melting and thermal dispersion effects on velocity profile for $Ra/Pe = F_r = N_r = 2, N_b = \gamma^* = Le = n = \Gamma = Ec = 1$

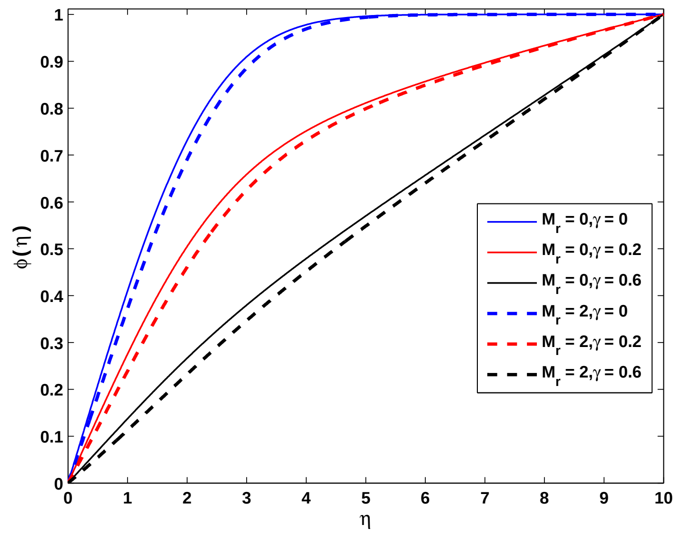

Fig. 5 Melting and thermal dispersion effects on velocity profile for $Ra/Pe = F_r = N_r = 2, N_b = Ec = \gamma^* = Le = n = \Gamma = 1$

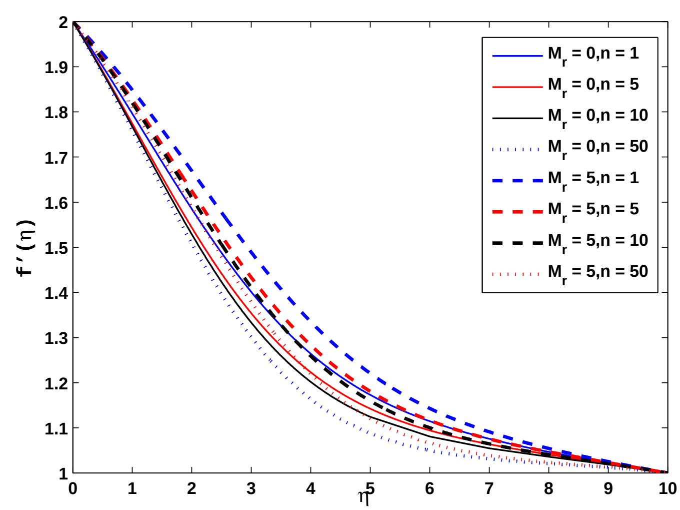

Fig. 6 Melting and order of chemical reaction effects on velocity profile for $Ra/Pe = F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = Ec = \Gamma = 1$

along with relevant boundary conditions

$$f(0) + 2M_r\theta'(0) = 0, \theta(0) = 0, \phi(0) = 0$$

and

$$f'(\infty) = 1, \theta(\infty) = 1, \phi(\infty) = 1 \tag{13}$$

where $Ra = \frac{kg\beta_T\rho(T_\infty - T_p)}{\nu\alpha_m}$, $Pe = \frac{u_\infty}{\alpha_m}$; Eckert number $Ec = \frac{u_\infty^2}{c_p\Delta T}$; non-Darcy parameter $F_r = \frac{2c_f\sqrt{k}u_\infty}{\nu}$; thermal dispersion parameter $\gamma = \frac{Ddu_\infty}{\alpha_m}$; buoyancy parameter $N_b = \frac{C_w - C_\infty}{T_p - T_\infty}$; radiation parameter $N_r = \frac{4\tilde{\sigma}T_p^3}{k_{et}k^*}$; Prandtl number $Pr = \frac{\nu}{\alpha_m}$; Lewis number $Le = \frac{\alpha_m}{D_m}$; solute dispersion parameter $\gamma^* = \frac{\beta du_\infty}{\alpha_m}$; modified chemical reaction parameter $\Gamma = \frac{K_n x(C_\infty - C_w)^{n-1}}{u_\infty}$ and melting parameter $M_r = \frac{c_f(T_\infty - T_p)}{L + c_s(T_p - T_0)}$. Nusselt and Sherwood numbers witness the nature of heat and mass transfer respectively in a fluid flow and are defined by

$$Nu_x = \frac{xq_w(x)}{k_{et}(T_p - T_\infty)} \tag{14}$$

$$Sh_x = \frac{x\widehat{m}(x)}{D_s(C_\infty - C_w)} \tag{15}$$

where

$$q_w(x) = -k_{et}\frac{\partial T}{\partial y}|_{y=0} \tag{16}$$

$$\widehat{m}(x) = -D_s\frac{\partial C}{\partial y}|_{y=0} \tag{17}$$

provide heat and mass flux at the wall respectively. Equations (8), (9), (14)–(17) yield the following dimensionless local Nusselt and Sherwood numbers respectively.

$$\frac{Nu_x}{\sqrt{Pe_x}} = -\left[1 + \frac{4}{3}N_r + \gamma f'(0)\right]\theta'(0) \tag{18}$$

$$\frac{Sh_x}{\sqrt{Pe_x}} = -\left[1 + \gamma^* f'(0)\right]\phi'(0) \tag{19}$$

3. NUMERICAL SOLUTION WITH MATLAB-BVP4C SOLVER

3.1. Introduction to bvp4c

BVP4C enforces a collocation method for the solution of a boundary value problem (BVP) of the form $y' = f(x, y, c)$ subject to general non–

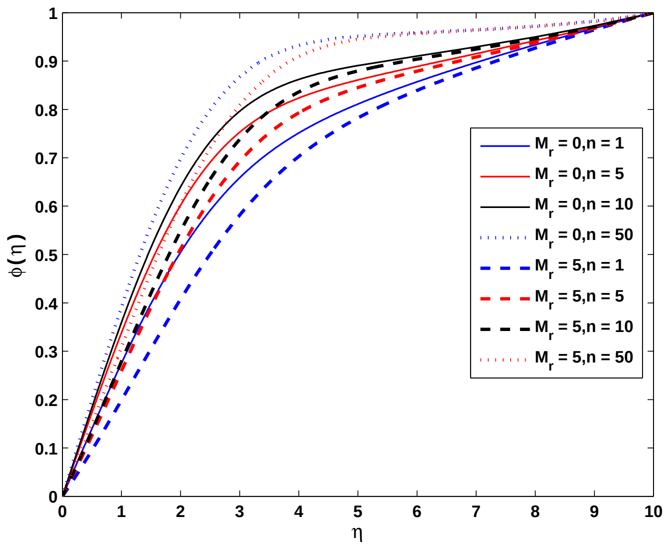

Fig. 7 Melting and order of chemical reaction effects on concentration profile for $Ra/Pe = F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = Ec = \Gamma = 1$

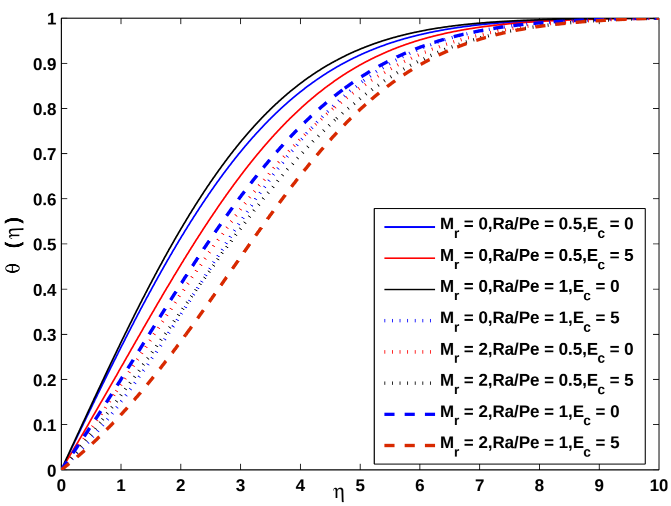

Fig. 8 Mixed convection along with Melting effects on temperature for $F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = n = \Gamma = 1$

linear, two point boundary conditions $g(y(a), y(b), c) = 0$ where $a \leq x \leq b$. The methodology involves the following two key stages(Jacek and Lawrence, 2001).

Stage1. Approximate solution S(x)

A cubic polynomial function S(x) is defined over each interval $[x_n, x_{n+1}]$ of each mesh $a = x_0 < x_1 < --- < x_n = b$, satisfying the boundary conditions $g(S(a), S(b)) = 0$. Also S(x) satisfies the collocates at both ends and the mid points of each sub interval. $S'(x_n) = f(x_n, S(x_n))$ $S'(\frac{x_n+x_{n+1}}{2}) = f\left[\frac{x_n+x_{n+1}}{2}, S(\frac{x_n+x_{n+1}}{2})\right]$ and $S'(x_{n+1}) = f(x_{n+1}, S(x_{n+1}))$. These are solved iteratively by Simpson's method. With modest assumptions, S(x) may be a fourth order approximation to an isolated solution y(x), $|||y(x) - S(x)||| \leq Ch^4$ where h is the maximum of step sizes $h_n = x_{n+1} - x_n$ and C is constant. Once S(x)is evaluated on a mesh with bvp4c, it can be found inexpensively at any x in [a,b] with 'bvpval' function.

Stage2. Residual function r(x)

To control the error, define the residual in ordinary differential equation as $r(x) = S'(x) - f[x, S(x)]$ with boundary condition $g(S(a), S(b))$.

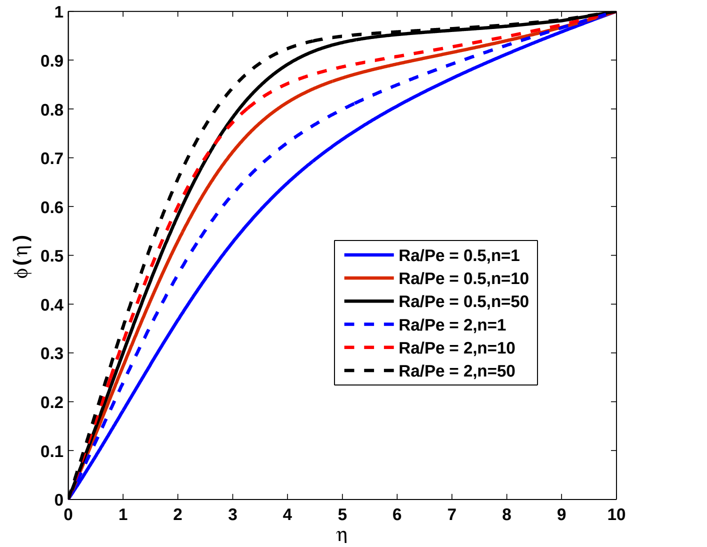

Fig. 9 Mixed convection and order of chemical reaction effects on concentration profile for $M_r = F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = Ec = \Gamma = 1$

If the residuals are uniformly small, S(x) is a good solution.

3.2. Numerical solution to boundary layer equations with bvp4c

The set of coupled non–linear ordinary differential equations (10)–(12) are reduced to a set of first order differential equations as follows:

$f_1' = f_2; f_2' = -\left(\frac{Ra}{Pe}\right)\left(\frac{f_4+N_b f_6}{1+F_r f_2}\right);$

$f_3' = f_4;\ f_4' = -\left(\frac{1}{1+\gamma f_2+(\frac{4}{3})N_r}\right)\left(\frac{1}{2}f_1 f_4 + \gamma f_2' f_4 + PrEcf_2^2\right);$

$f_5' = f_6;\ f_6' = -\left(\frac{Le\left[\gamma^* f_6 f_2' + \frac{1}{2}f_1 f_6 + \Gamma f_5^n\right]}{1+Le\gamma^* f_2}\right)$

where $f_1 = f, f_2 = f', f_3 = \theta, f_4 = \theta', f_5 = \phi, f_6 = \phi'$ Further the boundary conditions (13) are noted as $f_1(0) + 2M_r f_4(0) = 0, f_3(0) = 0, f_5(0) = 0$ and $f_2(\infty) = 1, f_3(\infty) = 1, f_5(\infty) = 1$ The absolute and relative errors of tolerance on the residuals are set by $options = bvpset('AbsTol', 1e-10, 'RelTol', 1e-10)$
Initial value problems are solved by 'bvpinit'
$solinit = bvpinit(linspace(0, 10, 10), [0\quad 10])$.
The following code executes the boundary value problem along with boundary conditions $sol = bvp4c(file\_ode, file\_bc, solinit, options)$
$dydx = [\quad y_2$
$-(Ra/Pe)*(y_4 + N_b*y_6)/(1 + F_r*y_2)$
y_4
$(-0.5*y_1*y_4+\gamma*y_4*(Ra/Pe)*(y_4+N_b*y_6)/(1+F_r*y_2)-Pr*Ec*((Ra/Pe)*(y_4+N_b*y_6)/(1+F_r*y_2))^2)/(1+\gamma*y_2+(4/3)*N_r)$
y_6
$(-0.5*y_1*y_6+\gamma^**y_6*(Ra/pe)*(y_4+N_b*y_6)/(1+F_r*y_2)+\Gamma*y_5^n)/(\gamma^**y_2+(1/Le))];$
Residual is calculated by the following syntax: $res = file\_bc(ya, yb)$

Advantages:

1. bvp4c is not a shooting method
2. The bound h hold for all x in $[a, b]$
3. Once S(x) is found on a mesh with bvp4c, it can be computed inexpensively at any value of x in $[a, b]$
4. bvp4c is based an algorithms that are credible even when the initial mesh is very poor, yet furnish the correct results as $h \to 0$
5. The solution y(x) is approximated over the whole interval $[a, b]$ and the boundary conditions are taken into account at all times, which is not in the case of shooting method.

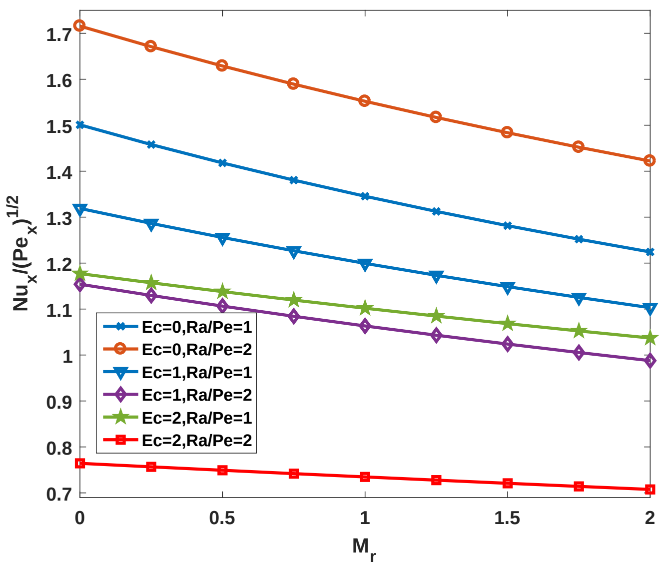

Fig. 10 Heat transfer against Melting parameter for $F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = n = \Gamma = 1$

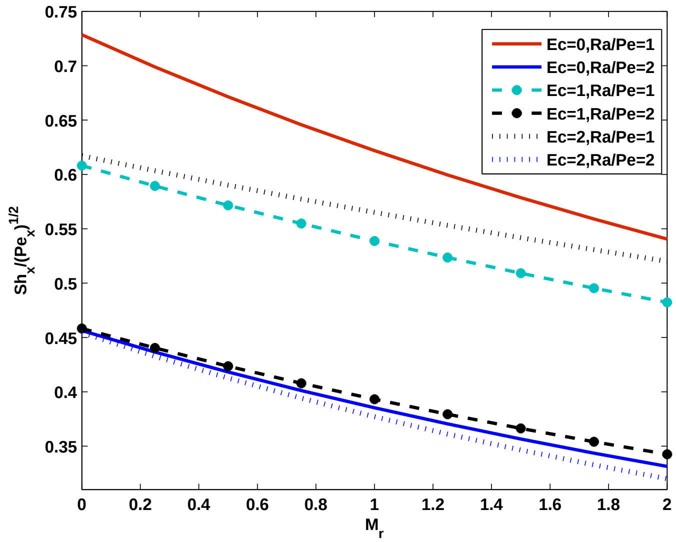

Fig. 11 Mass transfer against melting parameter for $F_r = N_r = 2, N_b = \gamma = \gamma^* = Le = n = \Gamma = 1$

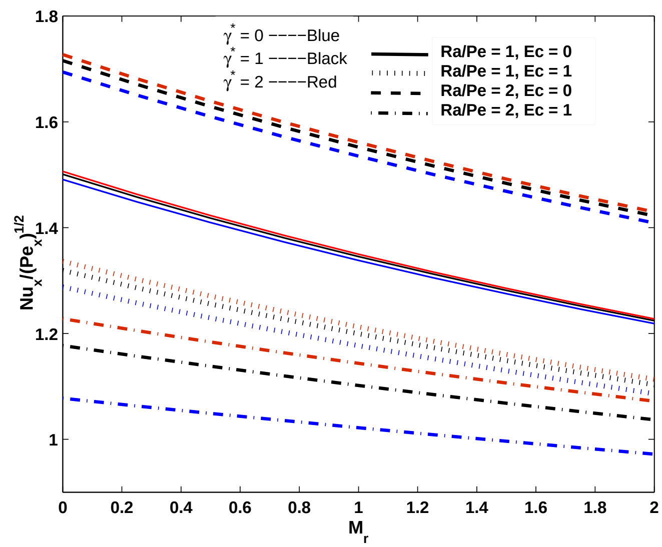

Fig. 12 Nusselt number against melting parameter for $F_r = N_r = 2, N_b = \gamma = Le = n = \Gamma = 1$

4. RESULTS AND DISCUSSION

In the present study, the physics of the problem is mainly focused on the combined effect of viscous dissipation and melting parameter on fluid characteristics. Prandtl number is fixed as 6.2 for comparative study. Fig. 2 depicts the impact of melting parameter together with Eckert number on velocity profile. It is observed that increasing the value of melting parameter shows an increment in both momentum boundary layer thickness and velocity profile. Further,for a fixed M_r, Eckert number enhances the velocity profile.

The theoretical phenomena is as follows: Kinetic energy will be converted to internal energy by work done against viscous stream representing viscous dissipation. The heat developed during this process enhances the velocity of the fluid particles. So more viscous dissipation triggers more velocity. From Fig. 3, descents are found in both thermal boundary layer and temperature profiles on increasing melting parameter. This may be due to the convective heat transfer is inhibited from liquid–saturated porous medium to the solid plate. A similar profile is observed in the case of Eckert number too. Effect of thermal dispersion on momentum and thermal boundary layers are notified in Fig. 4 and Fig. 5. Boundary layer thickness decreases with increase in the value of thermal dispersion. Also fluid particles attain momentum by increasing the dispersion. Further increasing thermal dispersion shows a decrement in fluid concentration level.

The influence of order of chemical reaction on velocity and concentration profiles are witnessed in Fig. 6 and Fig. 7. Increasing chemical reaction parameter reduces the fluid velocity and enhances the concentration profiles. Since it is known that higher order reactions will produce a kind of resistance and hence momentum will be reduced. Fig. 8 shows the effect of temperature on mixed convection along with viscous dissipation and melting parameter. Temperature is enhanced with increasing mixed convection parameter. The decrement in temperature profiles on enhancing Eckert number are more dominant in forced convection $(Ra/Pe = 1)$. In presence of melting these variations are getting lower.

Fig. 9 depicts the intensity of chemical reaction on concentration profiles at various convection parameters. It is noticed that increasing mixed convection parameter yields an increase in concentration profiles. Further an increase in order of reaction will cause more momentum in the molecules so that species will diffuse more rapidly yielding increase in concentration levels. The same can be viewed from this figure.

Fig. 10 renders the effect of melting on Nusselt number. It is evident from this figure that a fall in heat transfer occurs with increase in melting parameter. For a fixed M_r, it is observed that viscous dissipation resists the rate of heat transfer whereas mixed convection assists. Fig. 11 shows the effect of Sherwood number, a quantitative measure of mass transfer, on melting. In absence of melting$(M_r = 0)$, viscous dissipation has no effect on mass transfer in the case of mixed convection, but the same is notable for forced convection$(Ra/Pe = 1$, upper stream).

Sherwood number decreases with increasing melting parameter. Also we found the mass transfer is high in inviscid flows$(Ec = 0)$. The effect of solute dispersion on heat and mass transfer is described in Fig. 12 and Fig. 13 respectively. Nusselt number increases with increasing solute dispersion. This is very significant in a mixed convective viscous fluid $(Ra/Pe = 2, Ec = 1)$. Fig. 13 points out the solute dispersion enhances the local Sherwood number. It is observed from the same figure that this variation is more significant in presence of dispersion. Table1

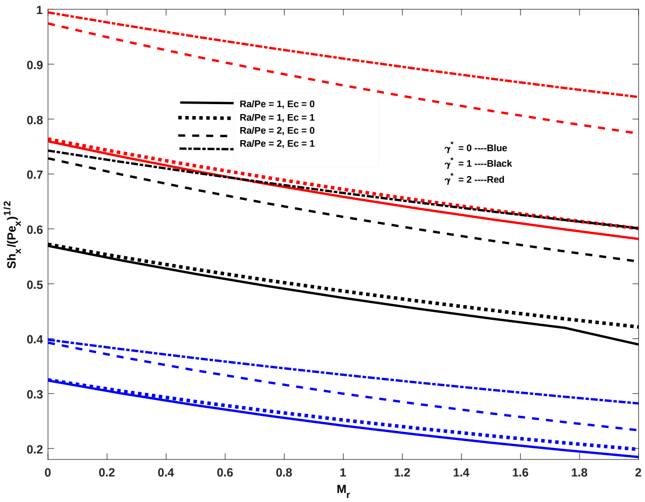

Fig. 13 Sherwood number against melting parameter for $F_r = N_r = 2, N_b = \gamma = Le = n = \Gamma = 1$

Table 1 Comparision of $f'(0)$with previous work for $F_r = N_r = N_b = \gamma = \gamma^* = Le = Ec = 0$

Ra/Pe	f'(0) (Hemalatha *et al.*, 2015)	f'(0)(present)
0.0	1.000	1.000
1.4	2.400	2.400
3.0	4.000	4.000
8.0	9.000	9.000
10.0	11.00	11.00
20.0	21.00	21.00

and Table2 show the agreement of the results with previous work in absence of viscous dissipation.

5. CONCLUSIONS

A two dimensional incompressible viscous fluid over a flat vertical plate is studied under chemical reaction and melting effect. The flow equations are numerically solved by means of MATLAB bvp4c solver. Results are obtained graphically. Melting causes an increase in velocity and decrease in temperature profiles. Viscous dissipation enhances velocity and reduces temperature profiles. Concentration decreases by increasing thermal dispersion. Solute dispersion increases both heat and mass transfer rates.

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NOMENCLATURE

C	Concentration of the fluid
c_f	Forchheimer constant
c_p	specific heat
c_s	Specific heat of solid phase
c_w	Concentration of the wall
D_d	Dispersion solutal diffusivity
D_m	Molecular solutal diffusivity
D_s	Thermal solute diffusivity
d	Particle mean diameter
g	Acceleration due to gravity
K_n	Chemical reaction rate of order n

Table 2 Comparision of $\theta'(0)$with previous work for $F_r = N_b = \gamma = \gamma^* = Le = Ec = 0$

Ra/Pe	$\theta'(0)$ (Hemalatha *et al.*, 2015)	$\theta'(0)$(present)
0.0	0.2706	0.2706
1.4	0.3801	0.3801
3.0	0.4745	0.4746
8.0	0.6902	0.6901
10.0	0.7594	0.7594
20.0	1.0383	1.0383

k	Permeability of porous medium
k_{et}	Effective thermal conductivity
k^*	Mean absorption coefficient
L	Latent heat of solid
Q_r	Radiative heat flux
Ra	Rayleigh number
Pe	Peclet number
T	Temperature of the fluid
T_p	Temperature of the plate
T_s	Temperature of solid phase far from interface
(u, v)	Velocity components along x,y axes respectively

Greek Symbols

α_T	Thermal diffusivity
α_m	Molecular diffusivity
α_d	Dispersion thermal diffusivity
β_T	Coefficient of thermal expansion
β_C	Coefficient of solute expansion
η	Similarity variable
ρ	fluid density
μ	dynamic viscosity
ν	Kinematic viscosity
$\widetilde{\sigma}$	Stefan Boltzman constant

Subscripts

∞	ambient enviroment

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