

# HEAT TRANSFER INFERENCES ON THE HERSCHEL BULKLEY FLUID FLOW UNDER PERISTALSIS

G. C. Sankad\*, Asha Patil

Research Centre (affiliated to VTU, Belagavi), Department of Mathematics,  
B.L.D.E.A.'s V.P. Dr. P. G. Halakatti College of Engineering and Technology,  
Vijayapur (Karnataka) 586101, India.

## ABSTRACT

Heat transfer effect on the flow of Herschel Bulkley fluid moving in a non-uniform channel is analyzed. The peristaltic wall is considered to be coated with a porous lining. The pertinent parameter effects are studied graphically for the analytical solutions of temperature profile, rate of temperature, heat transfer coefficient and mechanical efficiency. The temperature profile, heat transfer coefficient and the rate of temperature decrease with increase in the Darcy number. Thickening of the porous wall coating raises the temperature profile and the rate measure of temperature. Mechanical efficiency is more in a convergent channel than in uniform and divergent channels.

**Keywords:** *Non-uniform channel, Porous lining, Mechanical efficiency, Nusselt number.*

## 1. INTRODUCTION

Peristalsis is a coordinated action wherein, a contraction wave followed by a relaxation wave, pass through the channel. As they are propelled along, would always enter a segment which has actively relaxed and enlarged to receive them. Peristalsis is a significant mechanism which helps in the mixing and moving of fluids in human physiological system viz., passage of urine through the ureter, transportation of chime in the esophagus and in the gastro intestinal track. Peristaltic applications can also be found in mechanical devices in the transportation of corrosive materials, slurries and blood transfusion through the heart lung machine.

Latham's (1966) initial investigation on peristalsis paved way for many scholars to study and analyze the peristaltic motion (Burns and Parkes (1967), Shapiro (1967), Yin and Fung (1971)). Many authors including Tang and Fung (1975) have presented their study with the view that many physiological fluids as well as blood, flowing under peristalsis behave like a Newtonian fluid. Later on, it was concluded that under peristalsis, physiological fluids behave Newtonianly or non-Newtonianly. Kapur (1995) not only suggested the mathematical models for physiological flows by considering Newtonian as well as non-Newtonian fluids but also investigated blood theoretically considering it as both Newtonian and non-Newtonian fluid. Misery et al. (1996) explored the generalized Newtonian fluid motion through a channel, under peristalsis.

The peristaltic flow in porous tubes and channels has been under consideration due to their varied applications in the field of engineering and technology, to name a few: water percolation through soil, oil extraction and filtration from wells, sanitary engineering and irrigation, etc. Also, as it is understood that blood flows in small blood vessels is through peristalsis, movement of blood in blood vessels can be better understood by idealizing the blood vessels as channels with permeable walls.

To study the physiological parameters Tang and Fung (1975) assumed the physiological fluids to act as a Newtonian fluid. This advancement failed to explain the peristaltic method in tiny blood vessels and intestine, though it well explained peristalsis in ureter. Gopalan (1981) idealized each of the lung alveolar sheets as a channel covered by porous media to study the pulsatile blood flow. He analyzed that as blood flows in the lung is of low Reynolds number, a creeping flow is assumed in the channel.

Chaturani and Ranganathan (1993) have reported in their investigations that 'various ducts in living bodies are permeable'. Mishra and Ghosh (1997) mathematically modeled their observation of blood flow in tiny blood vessels in a porous channel. Shahawey and Sebai (2000) carried on their study through a porous tube. Ravikumar et al. (2011) explored the motion of a power law fluid moving under peristalsis, through an asymmetric channel having porous boundary walls. The forced convection in a circular tube filled with a Darcy-Brinkman-Forchheimer porous medium is analyzed by using spectral homotopy analysis by Rassoulinejad-Mousavi and Yaghoobi (2011). Rassoulinejad-Mousavi et al (2014) examined the effects of different wall boundary conditions on the flow of a Maxwell fluid through a porous saturated channel. Alsaedi et al. (2014) investigated the behavior of the couple stress fluid moving through a peristaltic porous medium.

Though it seems that the interaction of peristaltic flow of blood with heat transfer is not vital when we consider blood inside the body, it needs significance once it is drawn out of the body. Thus to bring out the importance of heat transfer in blood flow, assuming blood to be Casson fluid, Victor and Shah (1976) published their study on thermo-dynamical impacts on the transport of blood in a tube. Radhakrishnamacharya and Srinivasalu (2007) examined the wall property aspects on the peristaltic flow under heat transfer. Mekheimer and Elmabond (2008) gave a report of the consequences of heat transfer as well as the magnetic field, for the transport of a Newtonian fluid under peristalsis, through an annulus held vertical. Srinivas and Gayatri (2012) analyzed the peristaltic motion in a

\* Corresponding author Email : [math.gurunath@bldacet.ac.in](mailto:math.gurunath@bldacet.ac.in)

vertical asymmetric porous channel to analyze the heat transfer effects. Akbar et al. (2012) considered the flow through an asymmetric inclined channel with partial slip to analyze the heat transfer effect in the peristaltic transport. Applying two-equation energy model Rassoulinejad-Mousavi (2013) has studied the effects of heat transfer through a saturated porous channel by applying Homotopy analysis method. Dehia and Abdulhadi (2014) inspected the motion of Jeffery fluid in a porous channel and analyzed heat transfer and wall effects. They concluded that the temperature coefficient increases with the thermal conductivity.

The MHD transport of a Newtonian fluid under peristalsis was examined by Srinivas et al. (2009) to study the influences of both heat transfer and slip. The problem was also analyzed to study the compliant wall effects, flowing through a non-uniform porous channel. Rathod and Laxmi (2014) evaluated the effects of MHD and heat transfer on the peristaltic motion of Bingham fluid in a porous channel. Govindaraja et al. (2015) reported the impact of heat and mass transfer on the flow, generated by a train of sinusoidal wave of a couple stress fluid in a porous asymmetric channel. Ramesh and Devakar (2015) put forth their work on peristaltic motion of MHD non-Newtonian fluid in a vertical porous asymmetric channel to analyze heat and mass transfer effect.

Another non-Newtonian fluid the 'Herschel Bulkley' fluid is considered to be the more general fluid providing more accurate results than other forms of non-Newtonian fluids. The flow of Herschel Bulkley fluid through a channel under peristaltic motion is analyzed by Vajravelu et al (2005). Maruti Prasad and Radhakrishnamacharya (2008) studied the flow behavior of Herschel Bulkley fluid under multiple stenoses, considering an inclined tube. Maiti and Misra (2013) investigated the blood flow in micro vessels modeling blood to be Herschel Bulkley fluid. Sankad and Asha (2016) have studied the pumping impacts of different parameters on the flow of Herschel Bulkley fluid in a non-uniform channel with porous lining.

To study blood, preferably the Herschel Bulkley fluid model is suggested for its close behavior to blood and its flexibility to reduce into Bingham, Newtonian and Power law model. The heat transfer effect is studied under small Reynolds number and long wavelength assumptions, on the peristaltic motion of Herschel Bulkley fluid, through a non uniform channel. The influences of various parameters are compared through the graphs.

## 2. MATHEMATICAL MODELLING AND SOLUTION

### 2.1 Basic Equations

The governing equations of Herschel-Bulkley fluid flow as given by Tu and Deville (1996) are:

$$\nabla \cdot V = 0, \quad \nabla \cdot \sigma + \rho \frac{dV}{dt},$$

where  $\rho$  denotes the density,  $V$  the velocity and  $\sigma$  denotes the stress given as:

$$\begin{aligned} \sigma &= -pI + T, \text{ where} \\ T &= 2\mu D + S \text{ and} \\ S &= 2\eta D. \end{aligned}$$

Here the symmetric part of the velocity denoted by  $D$ , is given by:

$$D = \frac{1}{2}(L + L^T) \text{ and } L = \nabla V.$$

It is well known that, the material that flows only when the stress applied is more than a particular value, the yield stress of the material, is called visco plastic material. One such fluid that is considered for the

modeling in the present study is the Herschel Bulkley fluid. The behavior of the fluid is given by

$$\eta' = \frac{\tau_0 + f \left( \dot{\gamma}^n - \frac{\tau_0}{\mu_0} \right)^n}{\dot{\gamma}}$$

where  $n$ ,  $\tau_0$  and  $f$  respectively represent the power law index, the yield stress and the consistency factor.

### 2.2 Mathematical formulation

Herschel Bulkley fluid is considered to move in a non-uniform channel, whose walls are lined with a porous material, under heat transfer. A sinusoidal wave train is considered to travel along the elastic walls of the channel generating peristaltic motion in the fluid. The problem is discussed considering the half width of the channel. The plug flow region is the region between  $y = 0$  and  $y = y_0$ , where  $|\tau_{xy}| \leq \tau_0$ . In the region above the plug flow, between  $y = y_0$  and  $y = \eta$ ,  $|\tau_{xy}| \geq \tau_0$ .

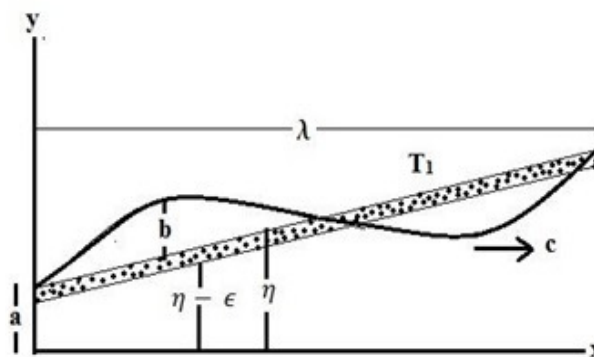


Fig. 1 Geometry of the flow.

The channel wall is given by the equation:

$$Y = \eta(X, t) = a' + b \sin \frac{2\pi X}{\lambda}, \quad (1)$$

where  $a' = a + kx$  and  $k$  denotes the dimensional non-uniformity parameter of the channel. The  $x$ -axis is considered horizontally along the center of the channel and perpendicular to it is considered the  $y$ -axis. The channel is of width  $'2a'$  and the wave is assumed to have amplitude  $'b'$ , wave length  $'\lambda'$  and wave velocity  $'c'$  at any time  $'t'$ .

Transforming from the laboratory frame  $(X, Y)$ , to the wave frame  $(x, y)$  with the relations:

$$\begin{aligned} x &= X - ct; & y &= Y; & u(x, y) &= U(X - ct, Y) - c; & v(x, y) &= V(X - ct, Y); \\ p(x) &= p(X, t). \end{aligned}$$

$(U, V)$  and  $(u, v)$  are the components of velocities in the laboratory frame and the wave frame respectively. Also  $'p'$  denotes the pressure in the wave frame. In several physiological circumstances we observe small Reynolds number, hence inertia free flow is assumed with infinite wavelength.

The dimensionless quantities involved are:

$$\begin{aligned} x' &= \frac{x}{\lambda}; & y' &= \frac{y}{a}; & t' &= \frac{ct}{\lambda}; & \eta' &= \frac{\eta}{a}; & \epsilon' &= \frac{\epsilon}{a}; & q' &= \frac{q}{ac}; \\ u' &= \frac{u}{c}; & v' &= \frac{v}{c\delta}; & \phi &= \frac{b}{a}; & \Psi' &= \frac{\Psi}{ac}; & Da &= \frac{s}{a^2}; & d' &= \frac{d\lambda}{a}; \\ \tau_0' &= \frac{\tau_0}{\mu \left( \frac{\epsilon}{a} \right)^n}; & F' &= \frac{fa}{\mu \lambda c}; \\ \text{also, } \theta &= \frac{T - T_0}{T_1 - T_0}; & Pr &= \frac{\rho v \xi}{s}; & Re &= \frac{\rho ca}{\mu}; & Ec &= \frac{c^2}{\xi(T_1 - T_0)}. \end{aligned}$$

Introducing the above quantities, the governing equations of motion are reduced to:

$$\eta(x) = 1 + kx + \phi \sin 2\pi x. \quad (2)$$

$$\frac{\partial}{\partial y}(\tau_{yx}) = -\frac{\partial p}{\partial x}, \quad (3)$$

where

$$\tau_{yx} = \left(-\frac{\partial u}{\partial y}\right)^n + \tau_0. \quad (4)$$

$$\frac{\partial^2 \theta}{\partial y^2} = \text{Br} \left[ \tau_{yx} \left(-\frac{\partial u}{\partial y}\right) \right], \quad (5)$$

where,  $\text{Br} = E_c P_r$ , is the Brinkman number,  $E_c$  is the Eckert number,  $P_r$  is the Prandtl number.

Further, the boundary conditions are:

$$\Psi = 0 \text{ when } y = 0. \quad (6)$$

$$\Psi_{yy} = 0 \text{ when } y = 0. \quad (7)$$

$$\tau_{yx} = 0 \text{ when } y = 0. \quad (8)$$

$$u = -\frac{\sqrt{Da}}{\alpha} \frac{\partial u}{\partial y} - 1 \text{ when } y = \eta(x) - \epsilon. \quad (9)$$

$$\frac{\partial \theta}{\partial y} = 0 \text{ when } y = 0. \quad (10)$$

$$\theta = 0 \text{ when } y = \eta(x) - \epsilon, \quad (11)$$

where  $u$  is the velocity,  $\alpha$  denotes the slip parameter,  $Da$  denotes the Darcy number,  $\tau$  represents the yield stress parameter,  $\theta$  denotes the temperature profile and  $\epsilon$  denotes the porous thickening of the wall. Also  $\Psi$  denotes the stream function with

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x}. \quad (12)$$

### 2.3 Mathematical solution:

Resolving Eqs. (3) and (4), together with the conditions (6) – (9) and (12), we obtain the velocity  $u$  as,

$$u = P^m \left[ \frac{1}{m+1} \{(n_2)^{m+1} - (y - y_0)^{m+1}\} + \frac{\sqrt{Da}}{\alpha} (n_2)^m \right] - 1, \quad (13)$$

where,

$$P = -\frac{\partial p}{\partial x}, \quad m = \frac{1}{n}, \quad n_1 = \eta - \epsilon \text{ and } n_2 = \eta - \epsilon - y_0 \quad (14)$$

In the plug flow region the velocity  $u_p$  is obtained as,

$$u_p = P^m (n_2)^m \left( \frac{n_2}{1+m} + \frac{\sqrt{Da}}{\alpha} \right) - 1. \quad (15)$$

At each cross section the volume flux 'q' is,

$$q = \int_0^{y_0} u_p dy + \int_{y_0}^{\eta-\epsilon} u dy, \text{ this gives,}$$

$$q = P^m \left[ \frac{(n_2)^{m+1}}{m+1} \left\{ n_1 - \frac{n_2}{m+2} \right\} + (n_2)^m \frac{\sqrt{Da}}{\alpha} (n_1) \right] - n_1. \quad (16)$$

From Eq. (16) we get,

$$P = -\frac{\partial p}{\partial x} = \left[ \frac{(q+n_1)(m+1)(m+2)\alpha}{(n_1)^{m+1}(1-\tau)^m \{ \alpha(h-\epsilon)(1-\tau)\{(m+2)-(1-\tau)\} + \sqrt{Da}(m+1)(m+2) \}} \right]^{\frac{1}{m}}. \quad (17)$$

At any instant the volume flow rate  $Q(X, t)$  is,

$$Q(X, t) = \int_0^H U(X, Y, t) dY. \quad (18)$$

Averaging Eq. (18) over a single period yields the volume flow rate  $\bar{Q}$  as,

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1. \quad (19)$$

Using Eq. (13),

$$\frac{\partial u}{\partial y} = -(Py - \tau_0)^m. \quad (20)$$

Substituting in Eq. (5), we get,

$$\frac{\partial^2 \theta}{\partial y^2} = \text{Br} \left[ \left(-\frac{\partial u}{\partial y}\right)^{n+1} + \tau_0 \left(-\frac{\partial u}{\partial y}\right) \right]. \quad (21)$$

Solving Eq. (21) along with the conditions (10) and (11)

$$\theta = \frac{c_3 (Py - \tau_0)^{m+2}}{(m+3)} [(Py - \tau_0)(m+1) + \tau_0(m+3)] + c_1 y + c_2, \quad (22)$$

where  $c_1$ ,  $c_2$  and  $c_3$  are given by,

$$c_1 = c_3 * (-\tau_0)^{m+2},$$

$$c_2 = \frac{c_3 [P * n_1 - \tau_0]^{m+2}}{(m+3)} [P(n_1)(m+1) + 2\tau_0] - c_1 n_1$$

and

$$c_3 = \frac{Br}{P(m+1)(m+2)}.$$

The coefficient of heat transfer is given by  $z = \eta_x \theta_y$ , which gives  $z$  as a function of  $x$ :

$$z = (c_3 * k_1) [(Py - \tau_0)^{m+1} (Py(m+1) + \tau_0) + (-\tau_0)^{m+2}], \quad (23)$$

where  $k_1 = k + 2\pi\phi \cos 2\pi x$

Also, the Nusselt number 'Nu' which measures the rate of the heat transfer is given by

$$Nu = -\theta_y, \text{ at } y = \eta - \epsilon. \quad (24)$$

Measure of the effectiveness of a machine in transforming the input energy and power into an output force and movement is called the mechanical efficiency. Mathematically, the mechanical efficiency is obtained by the relation:

$$E = \frac{\bar{Q} \Delta p}{\phi I}, \quad (25)$$

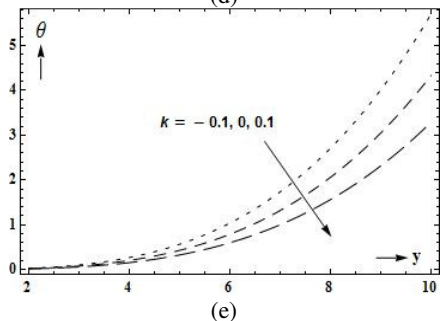
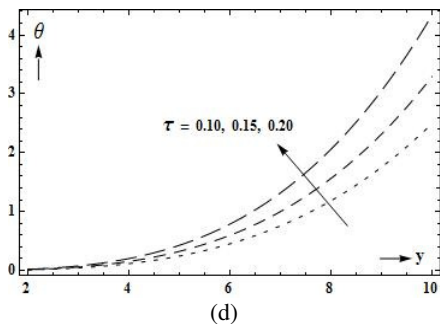
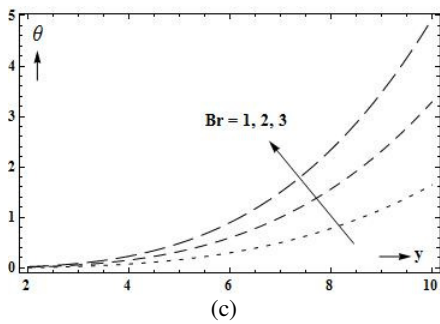
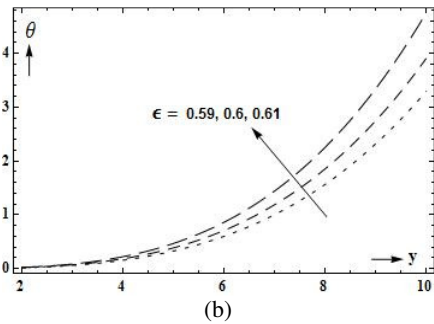
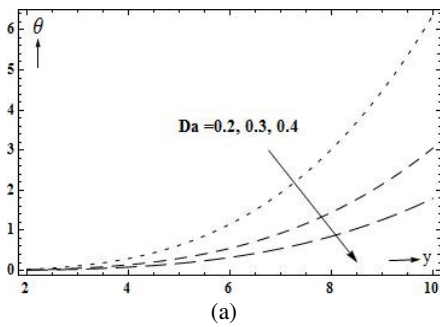
where,  $I = \int_0^1 \frac{\partial p}{\partial x} \sin(2\pi x) dx$ .

## 3. DISCUSSION OF RESULTS

### 3.1 Temperature profile:

From Eq. (22), the graphs of the temperature profile  $\theta$  against the radial coordinate  $y$  are plotted by means of Mathematica software and are displayed in Fig. 2. Effects of the parameters involved, Darcy number  $Da$ , porous thickening  $\epsilon$ , Brinkman number  $Br$ , yield stress  $\tau$  and non-uniform parameter  $k$  are analyzed for :  $n = 3$ ;  $k = 0.02$ ;  $a = 1$ ;  $\phi = 0.1$ ;  $\bar{Q} = 0.3$ ;  $\epsilon = 0.6$ ;  $\tau = 0.1$ ;  $\alpha = 0.3$ ;  $Da = 0.3$ ;  $z = 0.5$ ;  $Br = 2$ .

Figure 2(a) infers that rise in the Darcy number  $Da$  reduce the temperature. As the porous thickening  $\epsilon$  increases, the temperature also increases as shown in Fig. 2(b).

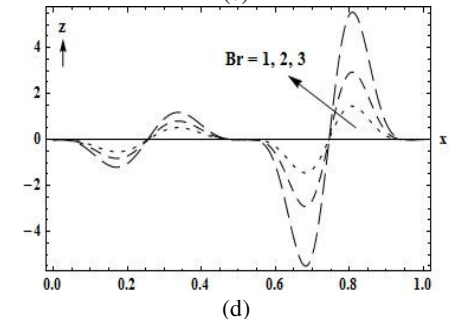
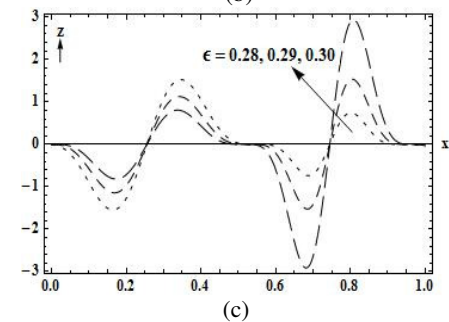
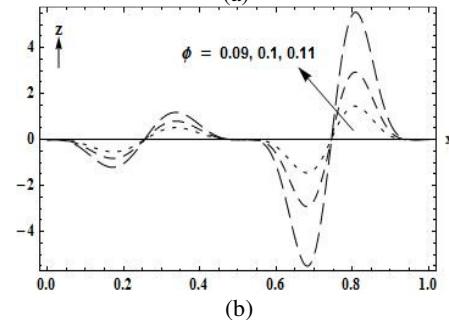
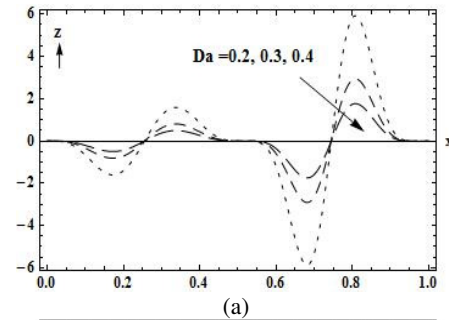


**Fig. 2** Graphs depicting variation of temperature profile  $\theta$  for variation of different parameters.

Also, increase in Brinkman number  $Br$  increases the temperature and is revealed from Fig. 2(c). Hina et al. (2012), Lakshminarayana et al. (2015), Ramesh and Devakar (2015) have also analyzed that temperature profile increases with the Brinkman number. The effect of variation in  $\tau$  is revealed from Fig. 2(d). It depicts that enhancing the values of  $\tau$  results in the rise of temperature. This result concurs with the results of Lakshminarayana et al. (2015) and Akbar and Butt (2015). From Fig. 2(e) it can be inferred that rise in temperature is more in a convergent channel than in uniform and divergent channels.

### 3.2 Heat transfer coefficient:

As a result of the peristaltic phenomenon, an oscillatory behavior of the coefficient of heat transfer is observed. Figure 3 elucidate the impact of various parameters on the coefficient of heat transfer  $z$ , figured out with the help of Eq. (23). The values taken for the parameters are:  $n = 3$ ;  $k = 0.02$ ;  $a = 1$ ;  $\phi = 0.1$ ;  $\bar{Q} = 0.3$ ;  $\epsilon = 0.3$ ;  $\tau = 0.1$ ;  $\alpha = 0.3$ ;  $Da = 0.3$ ;  $Br = 2$ .

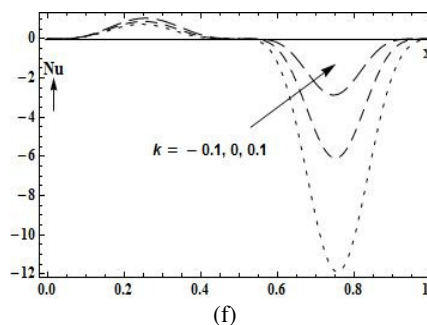
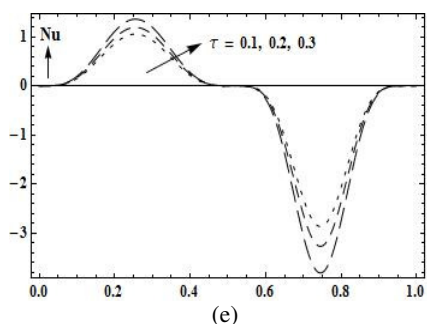
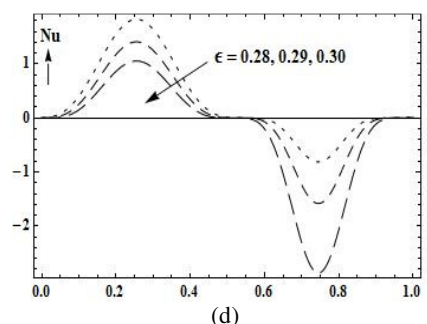
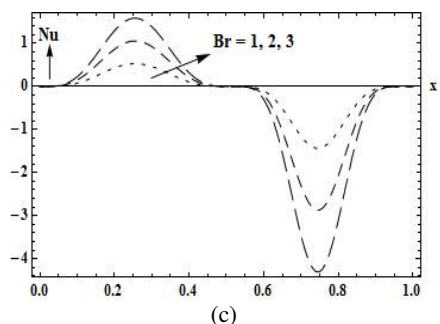
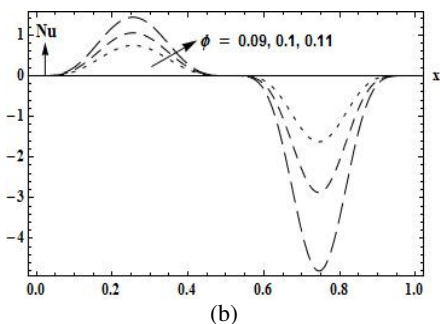
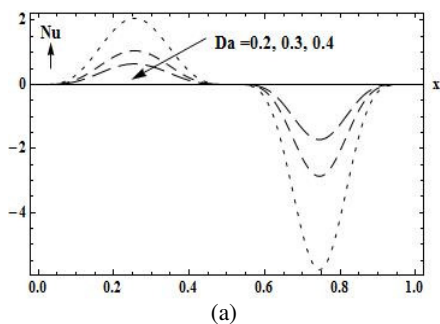


**Fig. 3** Graphs depicting variation of heat transfer  $z$  for variation of different parameters.

Figure 3(a) shows that, with rise in Darcy number  $Da$ , the magnitudinal value of the coefficient of heat transfer  $z$  decreases. The effect of  $\phi$  on the heat transfer coefficient is seen in Fig. 3(b). The graph says that the absolute value of  $z$  increases with  $\phi$ . It is clear from Fig. 3(c) that, as the porous thickening  $\epsilon$  increases, the magnitudinal value of heat transfer coefficient decreases up to  $x = 0.5$ , further it shows that with increase in the value of  $\epsilon$  there is increase in the absolute value of  $z$ . As the value of the Brinkman number increases the magnitudinal value of  $z$  also enhances as depicted in Fig. 3(d). Hina et al. (2012) and Kalidas Das (2012) are also of the same opinion for the results of Brinkman number.

### 3.3 Rate of heat transfer:

The Nusselt number ( $Nu$ ) value is to quantify the heat transfer rate. To observe the influence of the parameters  $Da$ ,  $\phi$ ,  $\epsilon$ ,  $Br$  and  $\tau$  on the rate of heat transfer, Eq. (24) has been evaluated numerically and the consequences are depicted through the graphs shown in Fig. 4. The values taken for the parameters involved are :  $n = 3$ ;  $k = 0.02$ ;  $a = 1$ ;  $\phi = 0.1$ ;  $\bar{Q} = 0.3$ ;  $\epsilon = 0.3$ ;  $\tau = 0.1$ ;  $\alpha = 0.3$ ;  $Da = 0.3$ ;  $y = 0.3$ ;  $Br = 2$ .



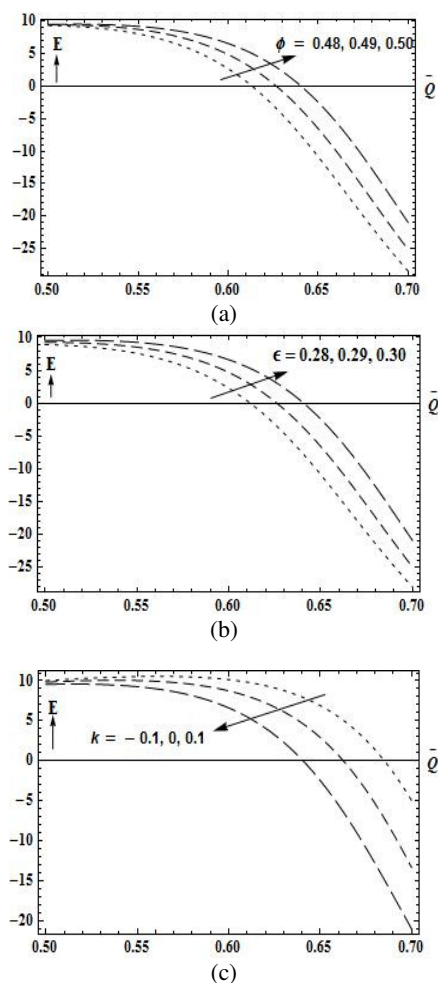
**Fig. 4** Graphs depicting variation of Nusselt number  $Nu$  for variation of different parameters.

It is clear from the graph in Fig. 4(a) that as the Darcy number  $Da$  is increased there is decrease in the absolute value of the Nusselt number. From Figs. 4(b), 4(c) and 4(e) it is seen that enhancing the values of amplitude ratio  $\phi$ , Brinkman number  $Br$  and the yield stress  $\tau$  respectively, the absolute value of the rate of heat transfer  $Nu$  rises. Figure 4(d) confirms that the heat transfer reduces as the porous lining  $\epsilon$  gets thicker. It is also inferred that the rate of heat transfer is more in a convergent channel when compared to a uniform and divergent channels from Fig. 4(f). The results obtained for Brinkman number, and the yield stress agree exactly with those of Lashminarayana et al. (2015).

### 3.4 Mechanical efficiency:

Figure 5, display the behavior of Mechanical efficiency with the variations in the different parameters of interest, after getting the analytical solution of mechanical efficiency as in Eq. (25), taking  $n = 3$ ;  $k = 0.02$ ;  $a = 1$ ;  $\phi = 0.5$ ;  $\epsilon = 0.3$ ;  $\tau = 0.1$ ;  $\alpha = 0.1$ ;  $Da = 0.3$ .

The graphs in Figs. 5(a), 5(b) and 5(c) correspondingly clarify that the Mechanical efficiency rises with the amplitude ratio  $\phi$  and also for the porous thickening  $\epsilon$  whereas the Mechanical efficiency  $E$  decreases with increases in the values of  $k$ .



**Fig. 5** Graphs depicting variation of Mechanical efficiency  $E$  for variation of different parameters.

Thus, Fig. 5(c) shows that the efficiency is more in a convergent channel than in uniform and divergent channels. Also it is noticed that the efficiency reduces with the time average velocity  $\bar{Q}$  for all the parameters.

#### 4. CONCLUSION

The heat effect on the flow of a Herschel Bulkley fluid under peristalsis is analyzed under the variations of the parameters involved with the considerations: (i) the channel is non-uniform; (ii) the elastic channel walls are lined with a non erodible permeable material. The analytical solutions for the temperature profile, heat transfer coefficient, rate of heat transfer and the mechanical efficiency are numerically analyzed through graphs, using the MATHEMATICA software.

This modeling may allow insight into the validity of reduction of the complications involved in modeling some non-Newtonian fluids like: transport of urine in the ureter and blood flow in the vessels beneath certain physiological conditions. The lumen of the coronary artery composed of fatty cholesterol and artery clogging blood clots may be viewed as porous medium. Thus the study may be helpful for the flow measurement through narrow arteries, formed due to the development of intravascular plaques, are helpful in investigating cardiovascular diseases. The interaction of heat transfer on the peristaltic motion has been of interest due to its relevance in hemodialysis and oxygenation.

#### NOMENCLATURE

|             |   |
|-------------|---|
| $a'$        | half width of the channel at any axial distance $x$ from the inlet. |
| $a$         | half width of the channel at the inlet                              |
| $b$         | amplitude of the wave   |
| $Br$        | Brinkman number   |
| $c$         | wave speed.   |
| $Da$        | Darcy number.   |
| $E$         | Mechanical efficiency   |
| $k$         | non-uniformity parameter  |
| $n$         | index parameter.  |
| $Nu$        | Nusselt number  |
| $p$         | pressure  |
| $P$         | expression defined in (14)  |
| $q$         | volume flux   |
| $Q$         | instantaneous volume flow rate                                      |
| $\bar{Q}$   | time average volume flow  |
| $t$         | time  |
| $T$         | period  |
| $u$         | axial velocity in the wave frame                                    |
| $U$         | axial velocity in the laboratory frame                              |
| $v$         | transverse velocity in the wave frame                               |
| $V$         | transverse velocity in the laboratory frame                         |
| $x$         | axial coordinate in the wave frame                                  |
| $X$         | axial coordinate in the laboratory frame                            |
| $y$         | transverse coordinate in the wave frame                             |
| $Y$         | transverse coordinate in the laboratory frame                       |
| $y_0$       | plug flow width   |
| $z$         | coefficient of heat transfer  |
| $\eta$      | expression defined by (1)   |
| $\alpha$    | slip parameter  |
| $\epsilon$  | porous thickening of the wall.                                      |
| $\theta$    | temperature profile   |
| $\phi$      | amplitude ratio   |
| $\lambda$   | wavelength  |
| $\tau$      | yield stress  |
| $\tau_0$    | yield stress in the plug flow region                                |
| $\tau_{yx}$ | shear stress.   |
| $\Psi$      | stream function   |
| $\Delta P$  | pressure difference   |

#### REFERENCES

- Akbar, N. S. and Adil Butt, 2015, "Heat Transfer Analysis for the Peristaltic Flow of Herschel Bulkley Fluid in a non-Uniform Inclined Channel," *Z. Naturforsch.*, **70**(1), 23–32.
- Akbar, N. S., Hayat, T., Nadeem, S. and Obaidat, S., 2012, "Peristaltic Flow of Williamson Fluid in an Asymmetric Channel with Partial Slip and Heat Transfer," *Int. J. of Heat and Mass Transfer*, **55**(7-8), 1855–1862.  
<https://doi.org/10.1016/j.ijheatmasstransfer.2011.11.038>
- Alsaedi, A., Ali, N., Tripathi, D. and Hayat, T., 2014, "Peristaltic Flow of Couple Stress Fluid through Uniform Porous Media," *Appl. Math. Mech.- Engl.*, **35**, 469–480.
- Burns, J. C. and Perks, J., 1967, "Peristaltic Motion," *J. Fluid Mech.*, **29**(4), 731–743.  
<https://doi.org/10.1017/S0022112067001156>

Chaturani, P. and Ranganathan, T. R., 1993, "Solute Transfer in Fluid Flow in Permeable Tubes with Application to Flow in Glomerular Capillaries," *Acta Mechanica*, **96**(1-4), 139–154.  
<https://doi.org/10.1007/BF01340706>

Dheia, G. and Abdulhadi, A. M., 2014, "Effects of Wall Properties and Heat Transfer on the Peristaltic Transport of a Jeffery Fluid through Porous Medium Channel," *Mathematical Theory and Modeling*, **4**, 86–99.

ElMisery, A. M., ElShehawy, E. F. and Hakeem, A. A., 1996, "Peristaltic Motion of an Incompressible Generalized Newtonian Fluid in a Planar Channel," *J. Phys. Soc., Japan*, **65**(11), 3524–3529.  
<http://dx.doi.org/10.1143/JPSJ.65.3524>

ElShehawy, E. F. and ElSebaei, W., 2000, "Peristaltic Transport in a Cylindrical Tube through a Porous Medium," *International Journal of Mathematics and Mathematical Sciences*, **24**(4), 217–230.  
<http://dx.doi.org/10.1155/S0161171200004737>

Gopalan, N. P., 1981, "Pulsatile Blood Flow in Rigid Pulmonary Alveolar Sheet with Porous Walls," *Bull Math Biol.*, **43**, 563–577.

Govindaraja, A., Siva, E. P. and Vidhya, M., 2015, "Combined Effect of Heat and Mass Transfer on MHD Peristaltic Transport of a Couple Stress Fluid in a Inclined Asymmetric Channel through a Porous Medium," *International Journal of Pure and Applied Mathematics*, **105**(4), 685–707.  
<http://dx.doi.org/10.12732/ijpam.v105i4.9>

Hina, S., Hayat, T., Asghar, S. and Hendi, A. A., 2012, "Influence of Compliant Walls on Peristaltic Motion with Heat/Mass Transfer and Chemical Reaction," *Int. J. Heat and Mass Transfer*, **55**(13-14), 3386–3394.  
<https://doi.org/10.1016/j.jheatmasstransfer.2012.02.074>

Kalidas Das, 2012, "Simultaneous Effects of Slip Conditions and Wall Properties on MHD Peristaltic Flow of a Maxwell Fluid with Heat Transfer," *Journal of Siberian Federal University. Mathematics and Physics*, **5**(3), 303–315.  
<http://elib.sfu-kras.ru/handle/2311/2918>

Kapur, J. N., 1995, *Mathematical Models in Biology and Medicine*, ISBN 81-8533682-2, Affiliated East West Press Ltd, India.

Lakshminarayana, P., Sreenadh, S. and Sucharitha, G., 2015, "The Influence of Slip Wall Properties on the Peristaltic Transport of a Conducting Bingham Fluid with Heat Transfer," *Procedia Engineering, International Conference on Computational Heat and Mass Transfer-2015*, **127**, 1087–1094  
<http://dx.doi.org/10.1016/j.proeng.2015.11.469>

Latham, T. W., 1966, *Fluid Motions in Peristaltic Pumping*. Master's Thesis, Massachusetts, MIT, Cambridge.  
<http://hdl.handle.net/1721.1/17282>

Maiti, S. and Misra, J. C., 2013, "Non-Newtonian Characteristics of Peristaltic Flow of Blood in Micro-Vessels," *Communications in Nonlinear Science and Numerical Simulation*, **18**(8), 1970–1988.  
<https://doi.org/10.1016/j.cnsns.2012.12.015>

Maruthi Prasad and K. Radhakrishnamacharya, G., 2008, "Flow of Herschel Bulkley Fluid through an Inclined Tube of non-Uniform cross-section with Multiple Stenoses," *Arch. Mech., Warszawa*, **60**, 161–172.

Mekheimer, Kh. S. and Abd.Elmaoud, Y., 2008, "The Influence of Heat Transfer and Magnetic Field on Peristaltic Transport of a Newtonian

Fluid in a Vertical Annulus: Application of an Endoscope," *Physics Letters A.*, **372**(10), 1657–1665.  
<https://doi.org/10.1016/j.physleta.2007.10.028>

Misra, J. C. and Ghosh, S. K., 1997, "A Mathematical Model for the Study of Blood Flow through a Channel with Permeable Walls," *Acta Mechanica*, **122**(1–4), 137–153.  
<https://doi.org/10.1007/BF01181995>

Radhakrishnamacharya, G. and Srinivasalu, Ch., 2007, "Influence of Wall Properties on Peristaltic Transport with Heat Transfer," *Comptes Rendus Mecanique*, **335**(7), 369–373.  
<http://dx.doi.org/10.1016/j.crme.2007.05.002>

Ramesh, K. and Devakar, M., 2015, "Effects of Heat and Mass Transfer on the Peristaltic Transport of MHD Couple Stress Fluid through Porous Medium in a Vertical Asymmetric Channel," *Journal of Fluids*, **2015**, 1–19  
<http://dx.doi.org/10.1155/2015/163832>

Rassouline-Mousavi, S. M., Abbas Bandy, "Analysis of Forced Convection in a Circular Tube Filled with a Darcy–Brinkman–Forchheimer Porous Medium using Spectral Homotopy Analysis method", *J. Fluids Eng.*, **133**(10), 101207–101216.  
<https://doi.org/10.1115/1.4004998>

Rassouline-Mousavi, S. M., 2013, "Heat Transfer through a Porous Saturated Channel with Permeable Walls using Two-Equation Energy Model", *Journal of Porous Media*, **16**(3), 241–254.  
<https://doi.org/10.1615/Jpormedia.V16.I3.60>

Rassouline-Mousavi, S. M., AbbasBandy, S. and Alsulami, H. H., 2014, "Analytical Flow Study of a Conducting Maxwell Fluid through a Porous Saturated Channel at Various Wall Boundary Conditions", *The European Physical Journal Plus*, 129:181  
<https://doi.org/10.1140/epjp/i2014-14181-4>

Rathod, V. P. and Laxmi, D., 2014, "Effects of Heat Transfer on the Peristaltic Mhd Flow of a Bingham Fluid through a Porous Medium in a Channel," *International Journal of Biomathematics*, **7**(6), 1450060–1450080.  
<https://doi.org/10.1142/S1793524514500600>

Ravi Kumar, Y. V. K., Krishna Kumari, S. V. H. N., Ramana Murthy, M. V. and Sreenadh, S., 2011, "Peristaltic Transport of a Power-Law Fluid in an Asymmetric Channel Bounded by Permeable Walls," *Pelgia Research Library Advances in Applied Science Research*, **2**(3), 396–406.  
<http://www.pelgiaresearchlibrary.com>

Sankad, G. C. and Asha Patil, 2016, "Effect of Porosity on the Peristaltic Pumping of a non-Newtonian Fluid in a Channel," *Journal of New Results in Science*, **10**, 1–9.

Shapiro, A. H., (1967), "Pumping and Retrograde Diffusion in Peristaltic Waves," *Proc. Workshop in Urethral Reflux in Children, Nat. Acad. Sci. Washington, DC*.

Srinivas, S. and Gayathri, R., 2009, "Peristaltic Transport of a Newtonian Fluid in a Vertical Asymmetric Channel with Heat Transfer and Porous Medium," *Applied Mathematics and Computation*, **215**(1), 185–196.  
<https://doi.org/10.1016/j.amc.2009.04.067>

Tang, H. T. and Fung, Y. C., 1975, "Fluid Movement in a Channel with Permeable Walls covered by Porous Media: A Model of Lung Alveolar Sheet," *J. Appl. Mech.*, **42**(1), 45–50.  
<https://doi.org/10.1115/1.3423551>

Tu, C. and Deville, M., 1996, "Pulsatile Flow of non-Newtonian Fluids through Arterial Stenosis", *J. Biomech*, **29** (7), 899–908.  
[https://doi.org/10.1016/0021-9290\(95\)00151-4](https://doi.org/10.1016/0021-9290(95)00151-4)

Vajravelu, K., Sreenadh, S. and Ramesh Babu, V., 2005, "Peristaltic pumping of a Herschel Bulkley Fluid in a Channel," *Applied Mathematics and Computation*, **169**(1), 726–735.  
<https://doi.org/10.1016/j.amc.2004.09.063>

Victor, S. A. and Shah, V. L., 1976, "Steady State Heat Transfer to Blood Flowing in the Entrance Region of a Tube," *International Journal of Heat and Mass Transfer*, **19**(7), 777–783.  
[https://doi.org/10.1016/0017-9310\(76\)90131-9](https://doi.org/10.1016/0017-9310(76)90131-9)

Yin, F. C. P. and Fung, Y. C., 1971, "Comparison of Theory and Experiment in Peristaltic Transport," *Journal of Fluid Mechanics*, **47**(1), 93–112.  
<https://doi.org/10.1017/S0022112071000958>