VARIABLE THERMAL CONDUCTIVITY INFLUENCE ON HYDROMAGNETIC FLOW PAST A STRETCHING CYLINDER IN A THERMALLY STRATIFIED MEDIUM WITH HEAT SOURCE/SINK

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ABSTRACT

This paper examines the variable thermal conductivity influence on MHD flow past a thermally stratified stretching cylinder with heat source or sink. The governing partial differential equations of the flow field are converted to a system of non-linear coupled similarity ordinary differential equations. Employing Shooting technique followed by Runge-Kutta method, the system is solved numerically. The effects of the various physical parameters countered in the flow field on the velocity, temperature as well as the skin friction coefficient and the rate of heat transfer near the wall are computed and illustrated graphically.

Keywords: Variable Thermal conductivity; Radiative heat transfer; Magneto hydrodynamic; Thermal stratification; Viscous flow.

1. INTRODUCTION


Stratification is due to the density variation in the fluid which gives rise to the temperature variations. So, that it is an important concept that one has to consider in studying the flow field. Best examples for this type of medium are congested containers, stratified ocean environment and biological chambers with heat blown walls. Flows with thermal stratification such as buoyant flow systems, Lake Thermohydraulics, volcanic flows and thermal treatment involved in the industrial processes. Stratification of the medium may arise due to a temperature variation, which gives rise to a density variation in the medium. This is known as thermal stratification and usually arises due to thermal energy input into the medium from heated bodies and thermal sources. The flow due to a thermally blown surface immersed in a stable stratified viscous fluid has been investigated experimentally. Deka and Neog (2009) explained in detail about the thermal stratification on an unsteady flow. Combined thermal and concentration stratifications on a Jeffrey fluid was discussed by Hayat et al. (2014). Nirmal (2016) analyzed the transient natural convection on a nanofluid with thermal stratification. Thermal stratification past an infinite cylinder was examined by Deka and Paul (2012). Kishore et al. (2010) investigated the effects of thermal stratification and dissipation along an infinite accelerated vertical plate.

Magnetic field effects on a conducting fluid received good attention from researchers. This is because hydromagnetic flow and heat transfer have become more important in industrially. For example, many metallurgical processes such as drawing, annealing and tinning of copper wires involve cooling of continuous strips or filaments by drawing them through a quiescent fluid. Controlling the rate of cooling in these processes can affect the properties of the final product. This can be done by using an electrically-conducting fluid and applying a magnetic field. Rushi Kumar (2015) explained the diffusion and radiation effects on plate in the presence of magnetic field. Mahapatra et al. (2014) analyzed the natural convection upon a horizontal flat plate with magnetic effects. The influence of heat absorption and source on MHD past stretching sheet in a non-Darcian medium was examined by Ibrahim and Shankar (2016). Poornima et al. (2016) analyzed the MHD mixed convective flow past a circular cylinder with chemical reaction effects. MHD convective flow past a linear stretching sheet using Lie symmetry analysis with thermal stratification was conducted by Rosmila et al. (2012).

The various physical properties of a fluid flow need not to be

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constant always, it may vary with respect to the time or temperature. Because of the temperature differences, there arises a physical property change. Slip flow of Casson fluid with variable fluid physical property under radiation was studied by Poornima et al. (2015). Bhaskar Reddy et al. (2014) investigated the impact of variable thermal conductivity past a stretching sheet. Subba Rao et al. (2016) studied the radiation effects past variable porosity with slip. On the vertical stretching sheet, the thermal buoyancy and variable thermal conductivity with heat source is studied by Abel et al. (2009). Ojjela et al. (2016) studied the varying thermal conductivity impact on UCM fluid. Rangi and Ahmad (2012) analyzed the boundary layer flow past a stretching cylinder and heat transfer with variable thermal conductivity.

With the above awareness, it is seen that the interaction of heat transfer with variable thermal conductivity in an electrically conducting fluid past a stretching cylinder in a thermally stratified medium has received a little attention. Hence, the object of this paper is to study the effect of variable thermal conductivity on MHD boundary layer flow past a stretching cylinder in a thermally stratified medium with heat generation or absorption.

2. MATHEMATICAL ANALYSIS

Consider a steady axi-symmetric flow along a continuously stretching circular cylinder. The fluid is electrically conducting, so that the magnetic field is applied transversely. The cylinder is completely immersed in thermally stratified saturated medium with a variable ambient temperature, where . The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is negligible. Figure 1 below gives the brief description of flow considered.

![Physical model and coordinate system](image)

Fig. 1 Physical model and coordinate system

The Prandtl’s boundary layer equations governing the flow under Boussinesq’s approximation are

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0
\]

(1)

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial^2 u}{\partial r^2} = \frac{\partial (ru)}{\partial r} + \frac{\partial }{\partial r} \left( r \frac{\partial u}{\partial r} \right) + g \beta (T - T_\infty) - \frac{\sigma B_i^2 u}{\rho}
\]

(2)

\[
u \frac{\partial T}{\partial x} + \nu \frac{\partial^2 T}{\partial r^2} = \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{Q_r}{\rho C_p} (T - T_\infty)
\]

(3)

The boundary conditions for the velocity and temperature fields are

\[u = U(x), \quad v = 0, \quad T = T_\infty(x) \atop u \to 0, \quad T \to T_\infty \atop \quad \text{as} \quad r \to \infty
\]

(4)

An equilibrium position is maintained between the convection fluid and the medium with respect to temperature. For liquid metals, the thermal conductivity varies linearly with temperature in the range from 0°F to 400°F. Therefore, we assume as

\[\alpha = \alpha_0 (1 + \epsilon \theta)
\]

(5)

where \(\epsilon (\leq 1)\) is the thermal conductivity parameter.

To get similarity solutions of equations (1) - (3) subject to the boundary conditions (4), we introduce the following similarity transformations.

\[
\eta = \frac{r^2 - R^2}{2R}, \quad \psi = \sqrt{U/r} x R(\eta), \quad \theta = \frac{T - T_\infty}{T_\infty - T_\infty}, \quad \frac{\partial}{\partial r}
\]

(6)

\[
\gamma = \frac{\sqrt{\nu L}}{R^2}, \quad \lambda = \frac{\eta \beta L}{U_\infty}, \quad M = \frac{\sigma B_i^2 L}{\rho U_\infty}, \quad \frac{\partial}{\partial r}
\]

(7)

In view of equation (6), equations (1), (2) and (3) take the following dimensionless form.

\[
(1 + 2 \eta^2) f'' + 2 \eta f' + \frac{f'''}{f} + \lambda \theta - M f' = 0
\]

(8)

where prime denotes the differentiation with respect to \(\eta\). The corresponding boundary conditions are

\[f'(\eta = 1) = 1, \quad f'(\eta = 0) = 0, \quad \theta(\eta = 0) = 1 - S
\]

(9)

The engineering quantities of our curiosity in this study are the skin friction or the shear stress coefficient \(C_f\) and the local Nusselt number \(N_u\), which are defined as

\[\tau_\infty = -\frac{\partial T}{\partial r} \bigg|_{r = r_\infty}, \quad \rho q_s = -k \frac{\partial T}{\partial r} \bigg|_{r = r_\infty}
\]

(10)

Using equation (9), the skin friction coefficient and local Nusselt number can be expressed as

\[\frac{1}{2} Re, C_f = f'(0), \quad \frac{N_u}{\sqrt{Re_s}} = -\theta'(0)
\]

(11)

where \(Re_s = \frac{U_\infty}{\nu}\) is the local Reynolds number.

3. NUMERICAL ANALYSIS

A closed form solution is not possible for the equations (7) - (8) and boundary conditions (9), as these are coupled and highly non-linear boundary value problem. It is employed shooting technique to convert the BVP into IVP and then the IVP is solved numerically by Runge-Kutta method. In shooting method, \(\eta_\text{max}\) (maximum \(\eta\) for which \(\theta(\eta) \to 0\) as \(\eta \to \infty\)) has to be shot first. For that the apt presumption \(\eta_\text{max} = 0\) is required so that the condition at the other end will be satisfied. To seek the missing \(\theta'(0)\), secant iterative method is applied.

After shooting the \(\eta_\text{max}\), it was chosen in such a way that further changes in it showed little changes (constant till \(10^{-5}\)) in the values of \(\theta'(0)\) vis-à-vis boundary condition \(\theta(\eta) \to 0\) as \(\eta \to \infty\) is to be shot first. For that the apt presumption \(\eta_\text{max} = 0\) is required so that the condition at the other end will be satisfied.

With an effective step size \(\Delta \eta = 0.05\) and a convergence criterion of \(10^{-5}\), a grid independent Runge-Kutta study was carried out by fixing \(\eta_\text{max} = 10\) (where it reaches the free stream velocity) to examine the solution in the quest for their optimization.

The shear stress coefficient and heat transfer rate at the wall are seen proportional to \(f'(\theta)\) and \(-\theta'(0)\), respectively. Their numerical computations are portrayed using graphs.

4. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in figures 2-17, to illustrate the influence of governing parameters viz., \(\gamma\) is the curvature parameter, \(\lambda\) is the mixed convection parameter, \(M\) is the magnetic parameter, \(S\) is the stratification parameter and \(Q\) is the heat generation or absorption parameter on the velocity and temperature. The value of the Prandtl number \(Pr\) is chosen to be 0.7, which corresponds air.
The present results are compared with those of Rangi and Ahmad (2012) for the appropriate reduced cases, and found that there is an excellent agreement, as presented in Table 1.

Table 1: Comparison of results $f''(0)$ for different values of curvature parameter $\gamma$ with $M = S = Q = \varepsilon = 0$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$f''(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-1.0000</td>
</tr>
<tr>
<td>0.25</td>
<td>-1.094378</td>
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<tr>
<td>0.5</td>
<td>-1.188715</td>
</tr>
<tr>
<td>0.75</td>
<td>-1.281833</td>
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<tr>
<td>1.0</td>
<td>-1.459308</td>
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</table>

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$f''(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-1.00006</td>
</tr>
<tr>
<td>0.25</td>
<td>-1.09438</td>
</tr>
<tr>
<td>0.5</td>
<td>-1.18869</td>
</tr>
<tr>
<td>0.75</td>
<td>-1.28180</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.45931</td>
</tr>
</tbody>
</table>

Fig. 2 Velocity profiles for different values of $M$

It is observed from figure 2, that the swiftness of flow gently reduces with the application transverse magnetic field which is due to a resistive type force called Lorentz force similar to drag force which in turn impedes the motion of the fluid flow and thus reducing its velocity. The velocity profiles reaches the free stream velocity at $\eta = 7$. The temperature profiles with two cases, constant and varying thermal conductivity (Figure 3). For, $\varepsilon = 0.1$, the temperature profiles are pronounced than $\varepsilon = 0$. Stratification also has significant influence on the temperature profiles.

Fig. 3 Temperature profiles for different values of $M$

From figure 4 it is clear that, the velocity of the flow field is reversing phenomena near the region $[0, 0.63]$ and at $\eta=0.63$, the profiles coincides then it approaches to zero within the region $[0.63, \infty)$. Velocity profile of the fluid increases with an increase in the curvature parameter $\gamma$. If $\gamma = 0$, the present geometry turns to flat plate.

Fig. 4 Velocity profiles for different values of $\gamma$

Figures 6 and 7 show for ascending $\lambda$ values, the buoyancy force becomes stronger, resulting in an increase of both the velocity and temperature profiles. Temperature is found to decrease with an increasing $Pr$.

Liquefied metals possess small Prandtl number usually varies of order 0.01~ 0.1 (from Bismuth (0.01) to Mercury (0.023)) and are preferred as coolants due to its high conductivity of heat. Such fluids have thicker thermal boundary layer. So, heat can diffuse from the surface faster than thinner boundary layers. Hence, Prandtl number controls the cooling rate in conducting flows.

Fig. 6 Velocity profiles for different values of $\lambda$

Fig. 5 Temperature profiles for different values of $\gamma$
The temperature of the fluid increases with an increase in heat generation or absorption parameter $Q$ (Figure 9). $Q = -0.1$ denotes the heat absorption. Positive values of $Q$ represent heat source. The presence of $\varepsilon$ has a significant impact on the fluid temperature.

Figure 10 depicts that an increase in $S$ results in decrease of the temperature, as the density variation will be less if one goes deeper and deeper inside the lake. Ambient thermal stratification shows considerable decrease the local buoyancy levels, which reduces the motion of the fluid.

Figure 11 illustrates the temperature rise with increasing variable thermal conductivity parameter $\varepsilon$ on the temperature. When $\varepsilon = 0$, the profile drawn will be with constant thermal conductivity, whereas the profiles rises for varying $\varepsilon$.

The variations of the velocity and temperature gradient at the surface are proportional to $f''(0)$ and $-\theta'(0)$. From Figure 12, it is clear that the surface velocity gradient decreases with an increase in $S$.

Also it is observed that the surface skin friction coefficient decreases with an increasing $M$. It is clear that (Figure 13), the wall shear stress decreases with $S$ and $\gamma$. Also friction is more at the surface of the cylinder ($\gamma = 0.5$) rather than the flat plate ($\gamma = 0$).
Fig. 13 Surface skin friction versus $S$ for different values of $\gamma$.

Fig. 14 Heat transfer rate versus $S$ for different values of $\gamma$.

The temperature gradient values are less with increasing $S$ and $\gamma$. For higher $Pr$ values, heat transfer rate is also high (Figure 15). The rate of heat transfer at the stretching surface decreases with an increase in $S$ for different $Q$ (Figure 16).

Fig. 15 Heat transfer rate versus $S$ for different values of $Pr$.

Figure 17 depicts the rate of heat transfer versus stratification parameter $S$, for different $\varepsilon$. It is observed that all profiles are converging at a common point $S = 1$ and the temperature gradient is 0.45.

5. CONCLUSIONS

A mathematical model has been framed to examine the influence of variable thermal conductivity on MHD flow past a stretching cylinder imbedded in a thermally stratified medium with internal heat source or sink. The following conclusions drawn are as follows:

- The curvature of stretching cylinder is an essential parameter, has considerable influence on motion of the flow and temperature field too.
- Due to the wall stretching, the skin friction coefficient decreases for both $\gamma$ and $M$.
- Presence of the buoyancy force decreases the magnitude of both the skin friction coefficient and heat transfer rate at the surface as the thermal stratification increases.
- The magnitude of both the skin friction coefficient and heat transfer rate at the surface are higher for a cylinder compared to a flat plate.
- For varying thermal conductivity, the curvature of the cylinder helps to enhance the heat transfer from the fluid.

NOMENCLATURE

- $B_0$: magnetic field
- $T_\infty$: ambient temperature
- $T_w$: wall temperature
- $R$: radius of the cylinder
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