HEAT GENERATION EFFECTS ON NATURAL CONVECTION IN POROUS CAVITY WITH DIFFERENT WALLS TEMPERATURE

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ABSTRACT

Natural convection heat transfer in a square cavity with a porous medium subjected to a uniform energy generation per unit volume is studied numerically in this paper. Temperature of the vertical walls is not equal but it is constant. There are two effective parameters in this condition that appear in the nondimensionalized equations and they are functions of temperature difference between hot and cold walls and energy generation in the porous medium. Nondimensionalized governing equations are obtained based on the Darcy model. A control volume approach is used for solving these equations. The effects of the variation of two governing parameters are investigated on the heat transfer rate, fluid flow and isotherms.

Key words: porous medium, natural convection, heat generation, cavity

1. INTRODUCTION

Porous media and the corresponding heat transfer phenomena has an important place in the industry. This importance has motivated many researchers to investigate the effective parameters on heat transfer in the porous media. Using porous media in the industry has many applications such as underground heat exchangers for energy saving, energy recovery and control of reactors temperatures, cooling electronic devices, thermal insulations, geothermal energy, etc. These and more applications are found in books of publishers such as Nield and Bejan (2006), Ingham et al. (2004), Ingham and Pop (2005) and Vafai (2000).

Heat transfer in a porous medium with different boundary conditions is one of important subjects considered in the literature. Anderson and Larit (1986) studied natural convection in a rectangular cavity with uniform wall temperatures where the vertical walls were cold, the bottom wall hot, and the upper wall adiabatic. Kimura and Bejan (1985) studied natural convection in rectangular chamber numerically, with cold temperature for bottom wall and constant heat flux for one of vertical wall. Pop and Saeid (2004) registered an article in porous medium field and showed increasing of Ra decreases time to achieve to constant Nusselt number. Chao and Ozoe (1983) analyzed natural convection in inclined cavity that half of bottom wall was adiabatic and second half had constant heat flux, and upper wall was cold. Granzarolli and Milanze (1995) reported results for cavity with cold temperature at vertical walls and constant heat flux at bottom side. Saeid (2007) presented a problem with both conduction and natural convection in porous medium that enclosed with finite thickness walls. Saeid and Pop (2005) have studied the natural convection in vertical porous medium with two heat source. They reported effects of Ra, Pe (Peclet number) and distance between heat sources on heat transfer. Chamkha et al. (2002) studied a porous cavity that is exposed to radiation heat transfer. Grosan et al. (2009) investigated the effects of magnetic field and heat generation on porous cavity and they presented results for effect of Ra and Ha number in heat transfer of cavity. Ha is a parameter that depends to magnetic field intensity.

Mealey and Merkin (2009) considered heat generation in porous mediums. They have shown that Edi vortexes created in center of vertical walls at large Ra number.

However, a very little works has been done for the case of heat generation effect on porous medium. The aim of this paper is to design different parameters and perform a extensive study in conditions that in addition to heat generation in porous cavity, The temperature of vertical walls is different.

2. GOVERNING EQUATIONS

A schematic diagram of simple porous medium with different boundary conditions and uniform heat generation per unit volume is shown in Fig. 1. The vertical walls are at constant temperature $T_1$ and $T_2$ and horizontal walls are adiabatic. In the porous medium Darcy’s law and Boussinesq approximation have been used and also fluid and solid are in thermodynamics equilibrium. The viscous drag and inertia terms in the governing equations are neglected which are valid assumptions for low Darcy and particle Reynolds numbers. With these assumptions, The Continuity, Darcy and energy equation for steady two dimensional flow in an isotropic and homogenous porous medium are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{gK\beta}{\nu} \frac{\partial T_p}{\partial x}
\]

\[
u \frac{\partial T_p}{\partial x} + v \frac{\partial T_p}{\partial y} = \alpha \left( \frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} \right) + \frac{q^*}{\rho C_p}
\]
Where the subscript \( p \) is for porous. These equations can be written in term of stream function \( \psi \) (defined as \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \)).

The value of non-dimensional stream function are zero on cavity’s walls:

\[
\Psi(0,Y) = \Psi(1,Y) = \Psi(X,0) = \Psi(X,1) = 0
\]  

and boundary conditions for non-dimensional temperature are:

\[
\frac{\partial \theta(X,0)}{\partial Y} = \frac{\partial \theta(X,1)}{\partial Y} = 0
\]

The physical quantities in this problem are local and average Nusselt number that defined in vertical walls as:

\[
\overline{Nu} = \int_0^1 Nu dy \quad Nu = -\frac{\partial \theta}{\partial X}
\]

3. NUMERICAL METHOD

Equations (5) and (6), with boundary conditions Eqs. (9-11), are integrated numerically by finite volume method (Patankar, 1966). The power law scheme is used for convection – diffusion terms in Eq. (6) and central difference approximation is used for diffusion terms of Eqs. (5) and (6).

For having accurate answers, it’s need to use an appropriate mesh scheme for geometry and boundary conditions. For mesh generation, different techniques can be used (Hoffmann, 1993). In present work algebraic method is used for mesh generation. But near of the boundaries, grid scheme is finer than central region because gradient of parameters in boundaries are bigger than central region. A used sample mesh scheme is shown in Fig. 2. For studying effect of grid scheme on results, variation of one of important parameters \( \Psi_{\text{max}} \) with mesh size has been shown in Fig. 3.
3.5. DISCUSSION AND RESULTS

5.1 Effects of Ra variation at constant \( \lambda_p \)

In first case at constant \( \lambda_p \), effect of Ra variations has been studied on the thermal fields, and circulation of fluid in the porous enclosure. Isotherm lines and stream lines are presented in fig (5) and also variation of \( \theta_{\text{max}} \) and \( \Psi_{\text{max}} \) with Ra is illustrated in fig (6).

It can be seen from figs (5 and 6) that by increasing of Ra, at constant \( \lambda_p \), \( \theta_{\text{max}} \) decreases and \( \Psi_{\text{max}} \) increases. This is due to increasing of convection heat transfer in porous enclosure that causes more cooling and increase of power of vortexes. For low Ra, Ra=10, at \( \lambda_p=100 \) two symmetry and similar vortexes has been created. However, by increasing of Ra the power of vortexes increases but counter clockwise vortexes are limited to top and left of cavity. Also it can be seen that isotherm lines are changed from almost elliptic lines to horizontal lines as shown in fig(5) that shows conversion of conduction heat transfer to convection.

Variation of the average Nusselt, \( \overline{Nu} \), has been illustrated with Ra in cold (cw) and hot wall (hw) in fig (7). By attention in fig (7) three points is considerable:

1. Increasing of average Nu in vertical boundries with increasing of Ra.
2. The average Nu is negative, \( Nu = -\frac{\partial \theta}{\partial X} \leq 0 \), at low Ra that is due to the over coming of the thermal field in the porous cavity to temperature gradient between vertical boundaries that leads to exiting of the heat flux from high temperature boundary such as low temperature boundary.
3. The difference between values of Nu in cold and hot wall with increasing of Ra is almost constant and equals to \( \lambda_p \). Therefore energy balances is as: \( \overline{Nu_{cw}} - \overline{Nu_{hw}} = \lambda_p \times H^2 \).

Table 1 \( \Psi_{\text{max}} \) and \( \theta_{\text{max}} \) in porous for this work and presented article by Grosan et al. (2009) at Ra=1000 and Ha=0.

<table>
<thead>
<tr>
<th>Ra = 1000</th>
<th>( \Psi_{\text{max}} )</th>
<th>( \theta_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grosan et al.</td>
<td>3.53</td>
<td>0.097</td>
</tr>
<tr>
<td>This work</td>
<td>3.51</td>
<td>0.098</td>
</tr>
</tbody>
</table>

4. VALIDATION

To ensure of the accuracy of this program, the results have been compared in special case in presented work by Grosan et al. (2009). This comparison is carried out in conditions that vertical walls temperature are equal, the horizontal walls are adiabatic and Ra=1000 and there is no magnetic field and heat is generating per volume unit in cavity. Stream lines and isotherm lines are presented for both works. in Figs. 4, a and b respectively. This comparison shows that stream lines and isotherm lines are so consistent. Numerical results for parameters \( \Psi_{\text{max}} \) and \( \theta_{\text{max}} \) are also given in Table 1.

Small difference between the results is due to different approximation and different convergence criteria during the solving process. After evaluation of applying computer code, results have been studied in two different cases, with Ra range \( (10 \leq Ra \leq 5000) \) and \( \lambda_p \) range \( (10 \leq \lambda_p \leq 100) \).
\[ \Psi_{\text{max}} = 3.17, \Psi_{\text{min}} = -3.84, \theta_{\text{max}} = 10.32 \]
\[ \text{(a) Ra} = 10 \]

\[ \Psi_{\text{max}} = 8.86, \Psi_{\text{min}} = -12.71, \theta_{\text{max}} = 5.21 \]
\[ \text{(b) Ra} = 100 \]

\[ \Psi_{\text{max}} = 18.39, \Psi_{\text{min}} = -36.85, \theta_{\text{max}} = 2.68 \]
\[ \text{(c) Ra} = 1000 \]

\[ \Psi_{\text{max}} = 25.03, \Psi_{\text{min}} = -76.31, \theta_{\text{max}} = 1.81 \]
\[ \text{(d) Ra} = 5000 \]

**Fig. 5** Streamlines (on the right), and isotherms (on the left) in simple porous cavity with heat generation at \( \lambda_p = 100 \) for temperature of vertical wall \( T_s \) and \( T_c \).

**Fig. 6** Variation of \( \theta_{\text{max}} \) and \( \psi_{\text{max}} \) with Ra at \( \lambda_p = 100 \)

**Fig. 7** Variation of \( \bar{Nu} \) in cold (Cw) and hot (Hw) walls with Ra at \( \lambda_p = 100 \)

### 5.2 Effects of \( \lambda_p \) variations at constant Ra

In the second case effect of variation of \( \lambda_p \) at constant Ra has been investigated. Schematic of isotherm and stream lines are shown in Fig (8) and variation of \( \theta_{\text{max}} \) and \( \psi_{\text{max}} \) are presented in fig (9) for Ra=1000.

As indicated in figs 8 and 9, increasing of \( \lambda_p \) that is equal to increasing of heat generation, causes heating of enclosure. Therefore power of vortexes increases and it can be seen that \( \theta_{\text{max}} \) and \( \psi_{\text{max}} \) increase too. By attention in fig 8, it is clear that in opposite of fig 5, that increasing of Ra at \( \lambda_p = 100 \) reduces area of positive vortexes and finally delete them. Increasing of \( \lambda_p \) at constant Ra creates positive vortexes and increase their area.

Also it can be seen in fig 8 that by increasing of \( \lambda_p \), isotherm lines nearly become horizontal that shows that power of convection heat transfer is increased. Variation of average Nu, with \( \lambda_p \), on cold and hot walls is shown in fig 10 for Ra=1000. As indicated in fig 10, however, with increasing of \( \lambda_p \), Nu increases in vertical boundaries but difference between Nu in cold and hot walls is equal to \( \lambda_p \) that shows energy balance in cavity. It is important to notice that negative Nu on hot wall is due to emersion of heat of this wall that is because of the thermal field in the cavity.
Natural convection heat transfer in cavity containing the porous medium with heat generation per unit volume is considered for numerical investigation. The governing parameters are Ra and $\lambda_p$, Ra is function of temperature difference of cold and hot boundaries. Parameter $\lambda_p$ is related to heat generation in cavity. The non-dimensional forms of the continuity, Darcy and energy equations are solved numerically. Parametric study is carried out and the results are presented to show the effect of these parameters on heat transfer and fluid flow characteristics. It is found that by increasing Ra at constant $\lambda_p$, the power of vortexes increases but maximum temperature of cavity, $\theta_{\text{max}}$ decreases. Also numerical results indicated that by increasing of $\lambda_p$ at constant Ra, both of $\theta_{\text{max}}$ and power of vortexes increases. Also, it is shown that by increasing Ra or $\lambda_p$ the $\overline{Nu}$ increases in cold and hot boundaries and energy balance is indefeasible as:

$$\overline{Nu}_{\text{cv}} - \overline{Nu}_{\text{hw}} = \lambda_p \times H^3$$

(14)

### 6. CONCLUSIONS

NOMENCLATURE

- $H$: height of cavity
- $g$: gravitational acceleration
- $R$: Rayleigh number
- $\lambda_p$: non-dimensional heat generation parameter
- $\overline{Nu}$: average Nusselt number
N \quad \text{local nusselt number}

K \quad \text{permeability of the porous medium}

k \quad \text{effective thermal conductivity of the porous medium}

q'' \quad \text{heat generation per unit volume in porous medium}

T_p \quad \text{temperature of porous medium}

T_c, T_w \quad \text{temperature of cold and hot wall}

U, V \quad \text{axial and radial velocities}

X, Y \quad \text{non-dimensional coordinates}

Greek Symbols

\alpha \quad \text{thermal diffusivity}

\theta \quad \text{non-dimensional temperature}

\nu \quad \text{kinematic viscosity}

\beta \quad \text{thermal expansion coefficient}

\psi \quad \text{stream function}

\Psi \quad \text{non-dimensional stream function}

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