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NUMERICAL ANALYSIS OF CASSON FERRO-HYBRID NANOFLUID FLOW OVER A STRETCHING SHEET UNDER CONSTANT WALL TEMPERATURE BOUNDARY CONDITION

Mohammed Z. Swalmeh*

Faculty of Arts and Sciences, Aqaba University of Technology, Aqaba 77110, Jordan

ABSTRACT

Heat transfer characteristics for free convection boundary layer flow with a Ferro-hybrid nanofluid in the Casson field, over a stretching sheet, have been numerically investigated and tested. The constant wall temperature boundary condition was applied in this study. The dimensional governing equations were transformed to partial differential equations (PDEs) and then solved numerically by an implicit finite difference scheme known as Keller box method. The Numerical findings were presented by tabular and figures by using MATLAB program. These numerical findings were gained according to considering and analyzing the impacts of Ferro-hybrid nanofluids Casson parameters, on the local skin friction coefficient and local Nusselt number, in addition, velocity and temperature. The study results elucidate that Ferro-hybrid nanofluid has the highest local Nusselt number than nanofluid, and it has lower velocity and temperature. In addition, the studied results were in excellent agreement with previously published results.

Keywords: *Ferro-hybrid nanofluid; Magnetohydrodynamic; Casson fluid; Stretching.*

1. INTRODUCTION

In recent years, a new concept has emerged, after nanofluid, namely "hybrid nanofluids", where the nanofluid was composed of a single nanoparticle suspended in a base fluid, to promote the heat transfer efficiency and physical properties for the base fluid, and the nanofluid was firstly introdused by Choi *et al.* (1995) Eastman *et al.* (2001). Also, nanofluid takes vastly survey study in mechanical and industrial sciences. Many modern articles have appeared for this nanofluid model, such that, Swalmeh *et al.* (2018), Dharmaiah *et al.* (2020), and Dharmaiah *et al.* (2018). The studied nanofluid boundary layer flow models are divided into two types, namely one-phase and two-phase model, which permanently are used to study the characteristics of nanofluids. Single-phase boundary layer flow gets the physical properties for nanoparticles with base fluid at the same time and deduced new enhanced physical properties as a single-phase blend with steady features (Abbas *et al.* (2020), Alkasasbeh *et al.* (2020), Hamarsheh *et al.* (2020)). Besides, the second model is the two-phase model which studies nanoparticle physical properties and their influences separately from the base fluid physical properties (Awais *et al.* (2016), Mehmood *et al.* (2017), Raju *et al.* (2018), Saleem *et al.* (2019)). To improve more eligibility for the nanofluid, hybrid nanofluids are the absorption of two various shapes of nanoparticles suspended in the regular fluid. Therefore, Hybrid nanofluid has remarkable advantages which don't subsist in the nanofluid. "hybrid nanofluids" are considered by Suresh *et al.* (2011), which are referenced to exhibit superior heat transfer characteristics and rheological demeanor along with got thermo-physical countenances. Hybrid nanofluid important implementations can be recognized in the systems that are generally utilized to transform solar energy to thermal energy are solar compilers, as well concentrators, besides that, another applications; domestic refrigerators, solar water warming, transformers, etc.Sidik *et al.* (2016). Further, some researchers have been published several numerical research papers concerning the boundary layer flow in a hybrid nanofluid over a stretching sheet, as Waini *et al.* (2019) Shoaib *et al.* (2020) Aly *et al.* (2019) Khashi'ie *et al.* (2020).

Ferro-nanofluid, or Ferro-hybrid nanofluid, is a colloidal magnetic nanofluid, with a size between 5 and15 nm, suspended in a carrier fluid, and coated with a surfactant layer. Ferrofluids have behavior like fluid magnets, therefore, in the non-existence of an exterior magnetic field, the nanoparticle's magnetic effects are randomly doled, hence, the liquid exhibits no clear magnetism Khashi'ie *et al.* (2020). Recently, heat transfer in convection boundary layer flow in the presence of Ferro-nanofluid has been taken into account, with magnetic fields, in fluid dynamics and engineering studies. Kandelousi (2014) investigated the influencing of spatially variable magnetic field, as well as rayleigh, magnetic, and hartmann numbers, on heat transfer Ferro-fluid flow, subject to constant heat flux boundary condition. A numerical investigation of the heat transfer improvements and fluid flow properties for a rotating cylinder with the effect of a magnetic dipole, as well as low Reynolds influence on numbers local Nusselt number, was considered by Selimefendigil *et al.* (2014). The convection heat transfer, by used Fe_3O_4 nanoparticle, suspended in based fluid, as a Ferro-hybrid nanofluid, was investigated by Moraveji *et al.* (2013). Considered the convection in Ferro-nanofluid flow of magnetic liquid over cylinder (see these related important studies; Li *et al.* (2017), Motlagh *et al.* (2020), and Khoshmehr *et al.* (2014)). Besides, modeled the convection heat transfer properties of Ferro-nanofluid on a stretching sheet (for additional specifics see articles; Khan *et al.* (2013), Andersson *et al.* (1998), Hussanan *et al.* (2018), and Bognár *et al.* (2020)).

At a medern time, various researches have been considered to study heat transfer in Casson fluid as non-newtonian fluid, and arrive at different grades of thermal competencies. The impacts of casson and

*Corresponding author Email: msawalmeh@aut.edu.jo

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magnetic parameters , as well as nanoparticle volume fraction parameter in convection boundary layer flow over sphere and cylinder was considered by Alwawi *et al.* (2019), Alwawi *et al.* (2020b), Alwawi *et al.* (2020a), Alzgool *et al.* (2019), Qadan *et al.* (2019). Besides, Haq *et al.* (2014) inspects the boundary layer flow in a Casson nanofluid on an exponentially permeable shrinking sheet with convective boundary condition, with magnetic field. Heat transfer properties and viscous dissipation in convection flow with Casson fluid at stagnation-point towards a stretching sheet was investigated by Mustafa *et al.* (2012). Also, heat transfer with casson fluid is considered in many different scientific problems, as Khoshrouye Ghiasi *et al.* (2018), Raju *et al.* (2017), and Awais *et al.* (2021). Also, Other remarkable investigations on Casson fluids include those by Ali *et al.* (2021), Kumar *et al.* (2021), and Rehman *et al.* (2021).

Motivating by all aforementioned above studies, the present study is utilizing (Keller box numerical method; Keller et al. (1970)) to investigate a free convection flow of Casson Ferro-hybrid nanofluids, as a single-phase model, under influence of magnetic field on a stretching sheet. the numerical results for the effect of Casson Ferro-hybrid nanofluid parameters on physical quantities, namely local skin friction and local Nusselt number, as well as temperature and velocity, are discussed via plotted and tabular.

2. MATHEMATICAL FORMULATIONS

In this study, the based Casson Ferro-hybrid nanofluids (Water/methanol), that contain (Al or Co)- Fe_3O_4 nanoparticles, were investigated, in incompressible MHD free convection boundary flow. The fluid flow was convected on a stretching sheet, such that the flow has started at y =0, and being settled in y >0. constant wall temperature (T_w>T_∞), as a boundary condition, was likewise considered, as shown in Fig. 1. The Ferro- hybrid nanofluid is penetrated in the Casson environment and a variable magnetic field intensity B_0 is used in the reverse orientation of the liquid flow. The flow is suggested to be in the x-direction, which extends along with the stretching sheet in the up direction and the y-axis is orthogonal to it. The effects of Ferro-hybrid nanofluid and Casson medium, as well as magnetic and electrical conductivity fields, have been inserted in the momentum equation.

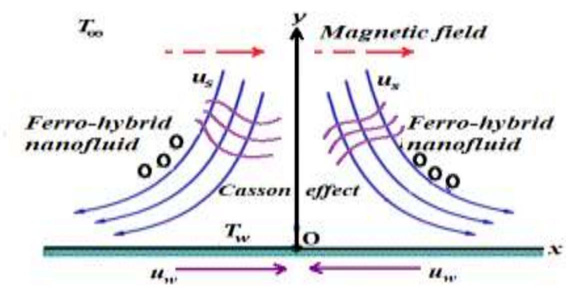

Fig. 1 A physical model of the problem.

The determined rheological characteristics of Casson fluid are gotten by The equation (see Alkasasbeh *et al.* (2020))

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y \big/ \sqrt{2\pi}\right) e_{ij} & \pi > \pi_c, \\ 2\left(\mu_B + p_y \big/ \sqrt{2\pi_c}\right) e_{ij} & \pi < \pi_c, \end{cases} \quad (1)$$

When $\pi > \pi_c$, we have

$$\mu = \mu_B + p_y \big/ \sqrt{2\pi} \quad (2)$$

After substitute the value of the yield stress of the fluid, $p_y = \mu_B \sqrt{2\pi} \big/ \beta$, we obtain as the following

$$\frac{\mu}{\rho} = \frac{\mu_B}{\rho} + \left(1 + \frac{1}{\beta}\right) \quad (3)$$

Here, $\pi = e_{ij}e_{ij}$, e_{ij} is called the *(i, j)*-th component of the deformation rate, π_c is defined a critical value of this product based on the non-Newtonian model, p_y, and μ_β are the yield stress of the fluid, and the plastic dynamic viscosity of the non-Newtonian fluid, respectively, As well as μ_B, ρ, and β are purely dependent on plastic dynamic viscosity, density, and parameter of the Casson fluid.

Consequently, relying on the all above supposition, the governing equations of the continuity, momentum and energy equations, for Casson Ferro-hybrid nanofluid on a stretching sheet can be gained as belows M. Z. Salleh *et al.* (2010) Manjunatha *et al.* (2019):

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \quad (4)$$

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \nu_{hnf}\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 \tilde{u}, \quad (5)$$

$$\tilde{u}\frac{\partial T}{\partial \tilde{x}} + \tilde{v}\frac{\partial T}{\partial \tilde{y}} = \frac{\kappa_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial \tilde{y}^2} \quad (6)$$

Subject to the utilized constant wall temperature boundary condition:

$$\tilde{u} = u_w(\tilde{x}) = a\tilde{x},\ \tilde{v} = 0,\ T = T_w \text{ at } \tilde{y} = 0,$$
$$\tilde{u} \to 0, T \to T_\infty \text{ as } \tilde{y} \to \infty, \quad (7)$$

where β_0^2 is the magnetic field strength, regarding for all other symbols and quantities are defined in nomenclature list. the nanofluid, α_{nf}, ρ_{nf}, $\left(\rho c_p\right)_{nf}$, μ_{nf}, k_{nf} and σ_{nf}, and hybrid nanofluid, α_{hnf}, ρ_{hnf}, $\left(\rho c_p\right)_{hnf}$, μ_{hnf}, k_{hnf} and σ_{hnf}, quantities, can be expressed in below displayed table 1.

Table 1: Thermo-physical properties (Alwawi *et al.* (2021))

Properties of nanofluid
$\rho_{nf} = (1-\chi)\rho_f + \chi\rho_s,$
$\left(\rho c_p\right)_{nf} = (1-\chi)\left(\rho c_p\right)_f + \chi\left(\rho c_p\right)_s,$
$\mu_{nf} = \frac{\mu_f}{(1-\chi)^{2.5}},$
$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_S + 2k_f) + \chi(k_f - k_s)},$
$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\sigma-1)\chi}{(\sigma+2)-(\sigma-1)\chi},\ \sigma = \frac{\sigma_s}{\sigma_s}$

(a)

Properties of hybrid nanofluid
$\rho_{hnf} = (1-\chi_2)[(1-\chi_1)\rho_f + \chi_1\rho_{s1}] + \chi_2\rho_{s2},$
$\left(\rho c_p\right)_{hnf} = (1-\chi_2)[(1-\chi_1)(\rho Cp)_f + \chi_1(\rho Cp)_{s1}] + \chi_2(\rho Cp)_{s2},$
$\mu_{hnf} = \frac{\mu_f}{(1-\chi_1)^{2.5}(1-\chi_2)^{2.5}},$
$\frac{k_{hnf}}{k_{bf}} = \frac{k_{s2} + 2k_{bf} - 2\chi_2(k_{bf} - k_{s2})}{k_{s2} + 2k_{bf} + \chi_2(k_{bf} - k_{s2})},$ $\frac{k_{bf}}{k_f} = \frac{k_{s1} + 2k_f - 2\chi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \chi_1(k_f - k_2)},$

$$\frac{\sigma_{hnf}}{\sigma_{bf}} = [\frac{\sigma_{s2} + 2\sigma_{bf} - 2\chi_2(\sigma_{bf} - \sigma_{s2})}{\sigma_{s2} + 2\sigma_{bf} + \chi_2(\sigma_{bf} - \sigma_{s2})}],$$
$$\frac{\sigma_{bf}}{\sigma_f} = [\frac{\sigma_{s1} + 2\sigma_f - 2\chi_1(\sigma_f - \sigma_{s1})}{\sigma_{s1} + 2\sigma_f + \chi_1(\sigma_f - \sigma_{s1})}]$$

(b)

Here, to convert the above dimensional governing equations (4) to (6), and boundary condition (7), to partial differential equations, we use the following similarity transformations and the physical properties in table 1. (see Salleh *et al.* (2010), Alkasasbeh *et al.* (2020)).

$$\tilde{u} = \frac{\partial \psi}{\partial \tilde{y}}, \tilde{v} = -\frac{\partial \psi}{\partial \tilde{x}},$$
$$\psi = (a\tilde{v})^{1/2}\tilde{x}f(\eta), \ \eta = (a\tilde{v})^{1/2}\tilde{y}, \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \qquad (8)$$

where ψ is the stream function.

Now, by substituting the equations (8) and the table 1 properties in equations (4) to (7), we gain the equations as below:

$$\frac{\rho_f}{\rho_{hnf}}\left(\frac{1}{(1-\chi_1)^{2.5}(1-\chi_2)^{2.5}}\right)\left(1+\frac{1}{\beta}\right)f''' + \qquad (9)$$
$$ff'' - (f')^2 - \frac{\rho_f}{\rho_{nf}}\frac{\sigma_{nf}}{\sigma_f}Mf' = 0$$

$$\frac{1}{\Pr}\left[\frac{k_{hnf}/k_f}{(1-\chi_2)[(1-\chi_1) + \chi_1(\rho Cp)_{s1}/(\rho Cp)_f] + \chi_2(\rho Cp)_{s2}/(\rho Cp)_f}\right]\theta''$$
$$+ f\theta' = 0, \qquad (10)$$

and the utilizing boundary condition will be written as

$$f' = 1, \ f = 0, \ \theta = 1 \text{ as } \eta = 0,$$
$$f' \to 0, \ \theta \to 0, \text{ as } \eta = \infty. \qquad (11)$$

Where χ_1 (Fe_3O_4) and χ_2 (Al or Co) are called nanoparticle volume fraction for (Co or Al) and Fe_3O_4, respectively, $\Pr = \frac{v_f}{\alpha_f}$ is the Prandtl number and $M = \left(\frac{\sigma_f B_0^2 a^2}{\rho_f v_f}\right)$ is called magnetic parameter.

In this study, our concern is localized on two physical quantities particularly, the local skin friction coefficient C_f and the local Nusselt number Nu, which are expressed by Alwawi *et al.* (2021):

$$C_f = \left(\frac{\tau_w}{\rho U_w^2}\right), \text{ and } Nu = \left(\frac{aq_w}{k_f(T_w - T_\infty)}\right), \qquad (12)$$

where, τ_w and q_w are shear stress, heat flux coefficient on the plane of the stretching sheet wall and are defined as below.

$$\tau_w = \mu_{nf}\left(\frac{\partial^2 u}{\partial \tilde{y}^2}\right)_{\tilde{y}=0} \text{ and } q_w = -k_{nf}\left(\frac{\partial T}{\partial \tilde{y}}\right)_{\tilde{y}=0}. \qquad (13)$$

substituting the equation (9) and (13), C_f and Nu be the following form

$$\mathrm{Re}^{1/2} C_f = \frac{1}{(1-\chi)^{2.5}}\left(1+\frac{1}{\beta}\right)f''(0), \ \mathrm{Re}^{-1/2} Nu = -\frac{k_{nf}}{k_f}\theta'(0), \qquad (14)$$

Such that, $\mathrm{Re}^{1/2} = (a\tilde{x}^2 / v_f)$ is signified the local Reynolds number.

3. RESULTS AND DISCUSSION

In this section, the numerical results of the magnetohydrodynamic effects and magnetic impacts in the presence of free convection of Ferro-hybrid nanofluids based on Methanol\water over a stretching sheet, are presented, as shown in bellow tables and figures. Besides, the constant wall temperature boundary condition is also considered. table 1 presents the thermo-physical characteristics of used nanoparticles, as well as water and methanol.

Table 2: Various mathematical values of thermophysical properties of nanoparticless of two base Casson Ferro-hybrid nanofluids, as Hamarsheh *et al.* (2020), Alwawi *et al.* (2021), and Mutuku-Njane (2014).

	Based fluids		Used nanoparticles	
Physical properties	Water	Methanol	Co	Al
k (W/mK)	0.613	0.2035	100	40
ρ(kg/m$^{3)}$	997.1	792	8900	3970
c_p (J/kgK)	4179	2545	420	765
σs(Sm^{-1})	5.5×10^{-6}	5×10^{-6}	1.602×10^7	1.12×10^5

Furthermore, the numerical comparison results for local skin friction and local Nusselt number, with different, Newtonian fluid, with recently published results for various values of Pr, are display in Tables 2 and 3. It is gotten that the current numerical outcomes of this study are in excellent agreement with the literature.

Table 3: Comparison of $\mathrm{Re}^{-1/2}Nu$ with different Newtonian fluid ($M = 0$, $\beta = \infty$, and $\chi_1 = \chi_2 = 0$), with several values of Prandlt number Pr.

Pr	$\mathrm{Re}^{-1/2}Nu$		
	Hassanien *et al.* (1998)	M. Z. Salleh *et al.* (2010)	Present
0.72	0.46325	0.46317	0.46322
1	0.58198	0.58198	0.58200
3	1.16525	1.16522	1.16523
5		1.56806	1.56810
7		1.89548	1.89551
10	2.30801	2.30821	2.30822
100	7.74925	7.76249	7.76253

Tables 4 display changes in the local skin friction and local Nusselt number of the hybrid Ferro-nanofluid and mono Ferro-nanofluid in the existence of the nanoparticles suspended in based fluids, with effects of Casson, nanoparticle volume fraction, and magnetic parameters. From these tables, it is noticed some observed physical properties variations between the mono nanofluids and the hybrid nanofluids, such that, Fe_3O_4\methanol, as nanofluids ($\chi_2 = 0$), have a higher local skin friction from (Al or Co)- Fe_3O_4\methanol, as hybrid nanofluid. Also, Fe_3O_4\methanol, as nanofluids, have lower local Nusselt number from (Al or Co)- Fe_3O_4\methanol, as hybrid nanofluid. on the other hand, the local skin friction and local Nusselt number for Co- Fe_3O_4 hybrid nanofluid are lesser than Al-Fe_3O_4 hybrid nanofluid, with impacts of different values of parameters χ_2, β, and M. Moreover, the water, as the based hybrid nanofluid, has a higher C_f and Nu than methanol. Further, when the parameter x increase, the local skin friction is decrease, but the local nusselt number increase, in presence of nanoparticles suspended in based fluid as Ferro-hybrid nanofluid. Besides that, if the parameter β and M increase, then the physical quantities Nu and C_f are decrease.

Table 4: Results of the skin friction coefficient C_f and local Nusselt number Nu, for a several values of Csson Ferro-hybrid nanofluid parameters, χ_2, β, and M, as well as $\chi_2 = 0.1$.

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β	χ_2	M	Co Fe_3O_4 Water C_f	Al Fe_3O_4 Water C_f	Co Fe_3O_4 Methanol C_f	Al Fe_3O_4 Methanol C_f	Fe_3O_4 Methanol C_f
3	0	0.5	-6.078	-6.078	-6.359	-6.359	-6.359
	0.03		-6.921	-6.602	-7.329	-6.949	
	0.07		-8.134	-7.375	-8.713	-7.817	
	0.1		-9.126	-8.020	-9.839	-8.539	
1	0	0.5					-2.596
1	0.1		-3.726	-3.274	-4.017	-3.486	
5	0						-10.055
5	0.1		-14431	-12681	-15.55	-13502	
10	0						-19.255
10	0.1		-27.633	-24.282	-29.791	-25.855	
3	0	0.1					-5.244
	0.1	0.1	-7.207	-6.333	-7.769	-6.743	
	0	1					-7.524
	0.1	1	-11.068	-9.436	-11.932	-10.356	
	0	3					-11.015
	0.1	3	-16.712	-14.685	-18.017	-15.637	

(a)

β	χ_2	M	Co Fe_3O_4 Water C_f	Al Fe_3O_4 Water C_f	Co Fe_3O_4 Methanol C_f	Al Fe_3O_4 Methanol C_f	Fe_3O_4 Methanol C_f
3	0	0.5	0.928	0.928	0.906	0.906	0.906
	0.03		1.070	1.080	1.041	1.056	
	0.07		1.296	1.317	1.254	1.289	
	0.1		1.495	1.523	1.453	1.492	
1	0	0.5					0.9654
1	0.1		1.588	1.598	1.544	1.581	
5	0						0.888
5	0.1		1.475	1.499	1.425	1.467	
10	0						0.873
10	0.1		1.452	1.479	1.401	1.453	
3	0	0.1					0.940
	0.1	0.1	1.574	1.592	1.519	1.558	
	0	1					0.850
	0.1	1	1.398	1.436	1.352	1.402	
	0	3					0.699
	0.1	3	1.147	1.200	1.091	1.165	

(b)

The impacts of nanoparticle volume fraction, Casson, magnetic parameters on temperature and velocity profiles are demonstrated in Fig.s 2 to 13. Fig.s 2 and 3 depict the abatement of temprature and velocity profiles, by boost the nanoparticles volume fraction. It is according to fact that the volume of the nanoparticle is inversely proportional to temperature and velocity.

Fig.s 4 to 7 explain that the increase of Casson and magnetic parameters, get observed raise in temperature and low in velocity profiles. Indeeded, a height in Casson and magnetic parameters is identifed by a descend in yield stress, hence lowering the velocity and highing in temperature. In addition, Fe3O4\methanol Ferro-nanofluid has a higher temperature and velocity than (Al, Co)\methanol Ferro-hybrid nanofluids. Also, the composition of Co- Fe3O4\methanol has a lower velocity and higher temperature than Al- Fe3O4\methanol as Ferro-hybrid nanofluid, with increase the several values of parameters, namely β, and M.

In Fig.s 8 and 13, we just focus on the comparison of the temperature and velocity distributions for Al- Fe_3O_4\water and Al- Fe_3O_4\methanol Ferro-hybrid nanofluids, with various values of x, B, and M. And we observed that Al- Fe_3O_4\methanol has a higher temperature than Al- Fe_3O_4\water. But the opposite case happens, Al- Fe_3O_4\methanol has a lower velocity than Al- Fe_3O_4\water, with an increase of the values of used parameters χ_2, β, and M.

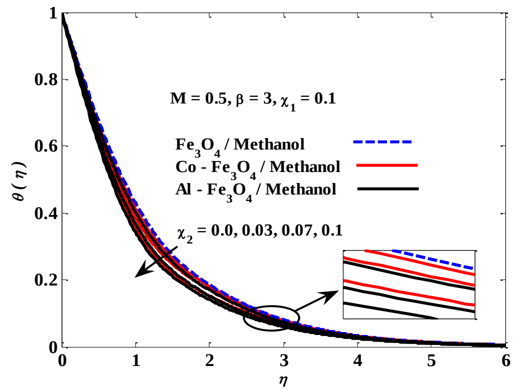

Fig. 2 Temperature against χ_2

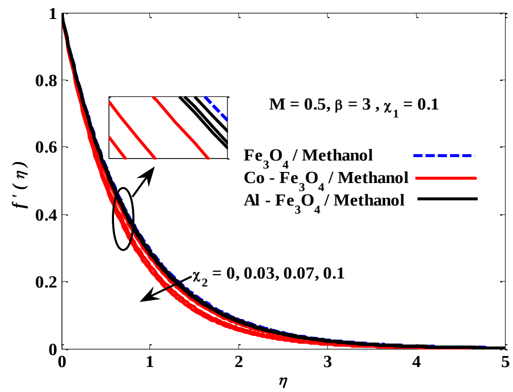

Fig. 3 Velocity profile against χ_2

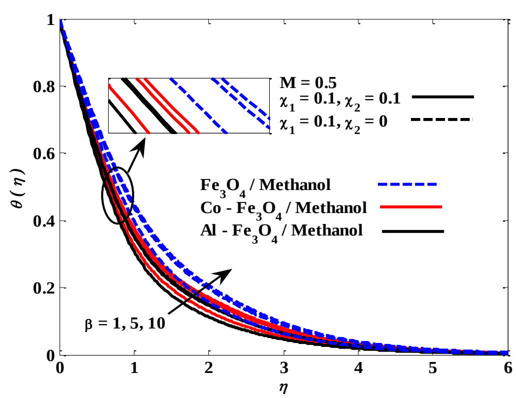

Fig. 4 Temperature against β

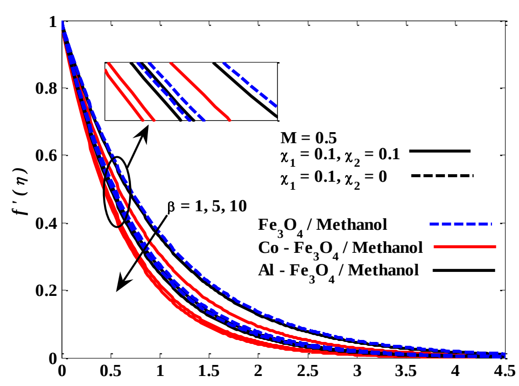

Fig. 5 Velocity profile against β

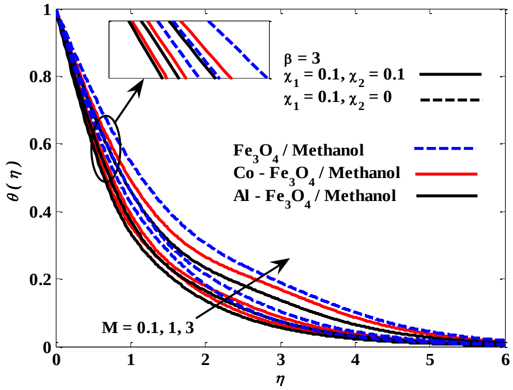

Fig. 6 Temperature against M

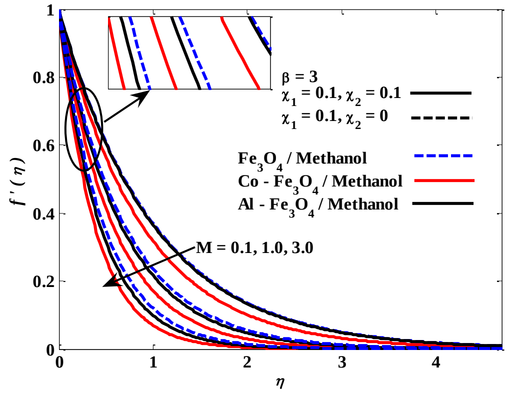

Fig. 7 Velocity profile against M

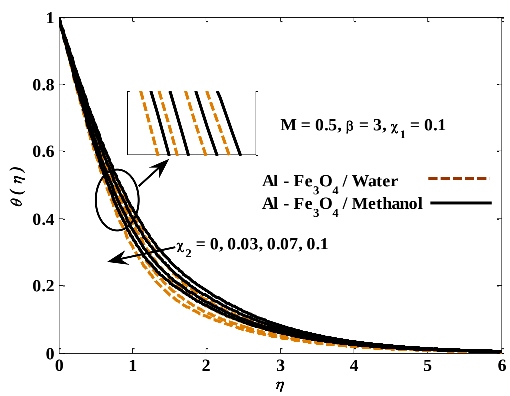

Fig. 8 Temperature against χ_2

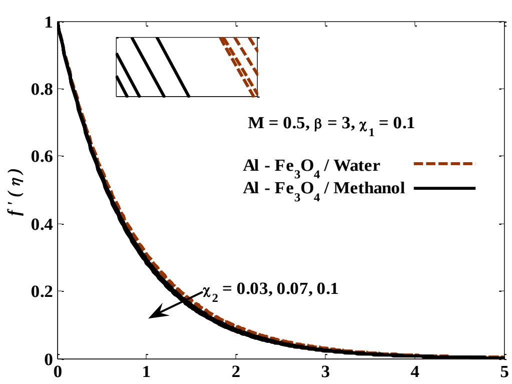

Fig. 9 Velocity profile against χ_2

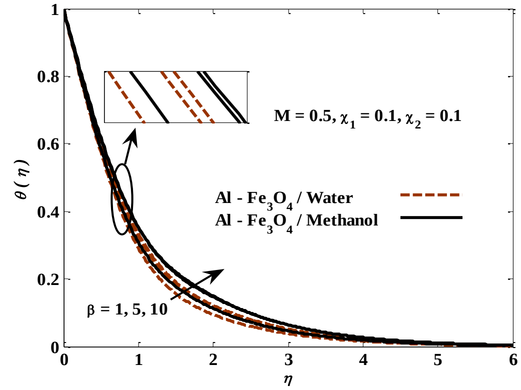

Fig. 10 Temperature against β

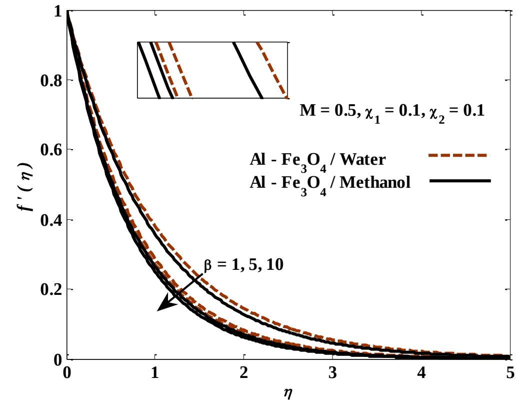

Fig. 11 Velocity profile against β

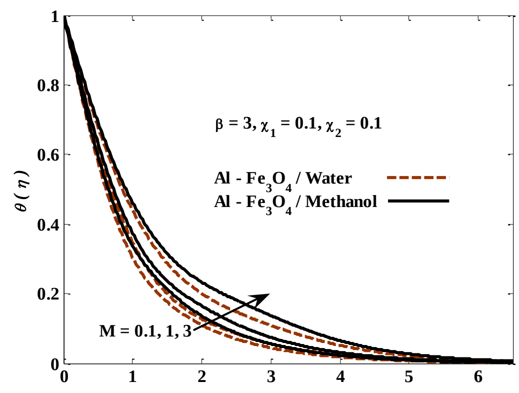

Fig. 12 Temperature against M

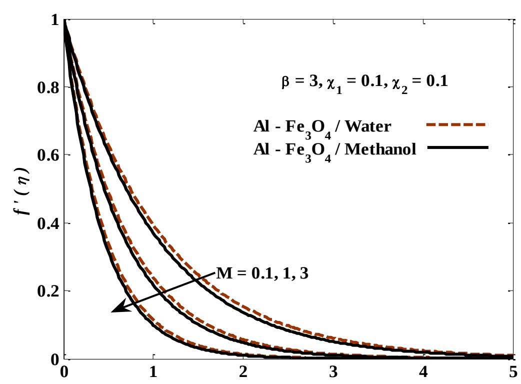

Fig. 13 Velocity profile against M

3. CONCLUSIONS

Heat transfer characteristics for the problem of Casson free convection flow of (Co, AL)-Fe_3O_4 suspended in water\methanol-based Ferro-hybrid nanofluid towards stretching sheet was numerically investigated in this paper. On the other hand, the impacts of magnetic field and constant wall temperature boundary conditions were also studied. Keller box numerical method was applied to solving the governing equations of the problem. The effect of concerned parameters on studied physical quantities was presented in the results and discussion section. From this investigation, the following significant points can be concluded:

1. Ferro-hybrid nanofluid has the highest *Nu*, and it has the lowest C_f, temperature, and velocity profiles compared with Ferro-nanofluid, regardless of the different values of the parameters affecting them.
2. Magnetic field strength and Casson parameter resulted in inhibiting velocity, C_f, and *Nu*, besides; it was go to raising the temperature. In addition, the nanoparticle volume fraction parameter got in curbing all physical quantities, namely velocity, temperature, and C_f, but, it was boost the *Nu*.
3. Water-based Ferro-hybrid nanofluid has the highest velocity profiles than methanol; but the opposite case happens, methanol-based Ferro-hybrid nanofluid has the highest temperature profiles than water, regardless of the several values of the parameters influencing them.
4. MHD Ferro-hybrid nanofluid flow is obviously impacted by the nanoparticle volume fraction, magnetic, Casson parameters. So, the consideration of this study supply additional acquaintance to the field of fluid mechanics

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NOMENCLATURE

B_0	magnetic field
k_f,	thermal conductivity of the based fluid
k_{nf},	thermal conductivity of the nanofluid
k_{hnf}	thermal conductivity of the hybrid nanofluid
c_p	specific heat
$(\rho c_p)_f$	heat capacity
$(\rho c_p)_{nf}$	heat capacity of the nanfluid
$(\rho c_p)_{hnf}$	heat capacity of the hybrid nanofluid
M	magnetic parameter
q_w	heat flux coefficient
Re	Local Reynold’s number
u_s	stream velocity
u_w	stretching velocity
T_w	wall temperature
T_∞	Temperature of the ambient fluids
x, y	coordinates (m)
Pr	Prandtl number
Greek Symbols	
χ	nanoparticle volume fraction parameter
η	dimensionless similarity variable
ψ	stream function
ρ	density
μ_{nf}	dynamic viscosity of the nanofluid
μ_{hnf}	dynamic viscosity of the hybrid nanofluid
θ	temperature
Subscripts	
nf	nanofluid
hnf	hybrid nanofluid
∞	ambient environment

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