



INVESTIGATION ON CNTS-WATER AND HUMAN BLOOD BASED CASSON NANOFLUID FLOW OVER A STRETCHING SHEET UNDER IMPACT OF MAGNETIC FIELD

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ABSTRACT

This study aims at considering the properties of heat transfer and magneto-hydrodynamics (MHD) Casson nanofluid at the existence of free convection boundary layer flow with Carbon Nanotubes (CNTs) suspended in human blood/water as based fluid on a stretching sheet. Two types of CNTs nanoparticles, single walled carbon nanotubes (SWCNTs) and multi walled carbon nanotubes (MWCNTs), are taken into account. The governing partial differential equations are transformed to partial differential equations using similar transformation, then solved numerically by an implicit finite difference scheme known as Keller-box method (KBM). The results for physical quantities, the local skin friction, and local Nusselt number, as well as temperature and velocity, are discussed under the magnetic nanofluid Casson parameters. This work is compared with recently published results on the Newtonian fluid as a special case and shows very good agreement.

Keywords: Casson Nanofluid fluid; MHD, Stretching Sheet; SWCNTs and MWCNTs.

1. INTRODUCTION

Casson fluid is a subcategory of Non-Newtonian fluids as toothpaste, food materials, and plastic has several utilizes in medical chemical industries and cosmetics. It is hard to handle this type of fluid, because it behaves as an elastic solid, such that no flow happens with short shear stress. Hence, different fluid models are showed to characterize the effectiveness of the materials. In the current consideration, we investigate the Casson model. This model was first conducted by Casson (1959). After that, many types of researches had been conducted in the same field. Mustafa *et al.* (2012); Mustafa *et al.* (2011) investigated the parallel free stream in the presence of the Casson fluid on a semi-infinite flat plate and in the region of stagnation point towards a stretching sheet. Nadeem and UI (2013) studied the boundary layer three- dimensional flow of Casson fluid on a stretching sheet. Another Various researches on Casson fluids can also be found in the literature of Hayat *et al.* (2012), Nadeem *et al.* (2013), Bhattacharyya (2013); Qing *et al.* (2016), Tahir *et al.* (2017), Alsagri *et al.* (2019), Hussanan *et al.* (2018), and Alwawi *et al.* (2019).

Recently, several researches have been conducted to develop the heat transfer and reach various levels of thermal competences. This technique can be used through increasing the thermal conductivity. Choi and Eastman (1995) were the first who examined the enhancement of heat transfer in convectonal fluids with nanoparticles, typically with a size less than 100 nm. After that, many researchers considered it because of its thermal characteristics and its future applications. Moreover, mathematical models have been conducted to investigate in the properties of nanofluids Tiwari and Das (2007). Tiwari-Das model

has succeeded in describing the behaviors of nanofluid in boundary layer flow. Besides, many researchers published various articles around the boundary layer flow in a nanofluid, such as Chamkha and Aly (2010); Chamkha *et al.* (2011), Bhattacharyya (2013); Chamkha (2011), Mukhopadhyay *et al.* (2013) and Swalmeh *et al.* (2019 a,b).

It is said that human blood is an example of Casson fluid, and many studies used physical properties of the human blood as based fluid under boundary layer flow, see Khalid *et al.* (2018), Rashidi *et al.* (2016), and Alsagri *et al.* (2019). CNTs (carbon nanotubes) have a long cylindric shape, such as structures of carbon atoms with sizes typically of 0.7-50 nm. In 1991, the concept of CNTs was first investigated by Iijima (1991). Carbon nanotubes are split into two types, namely single wall carbon nanotubes (SWCNTs), and multi wall carbon nanotubes (MWCNTs), relying upon concentric layers of rolled graphene lamina. Moreover, carbon nanotubes are expected to be a creative matter on our time because of their industrial and mechanical applications. There are many studies that take the CNTs nanoparticles into consideration. Haq *et al.* (2015) considered the effects of the thermo-physical characteristics of CNTs nanoparticles based nanofluids on a stretching surface. The heat transfer flow of nanofluids including CNTs on a flat plate subjected to the Navier slip boundary condition, and the impacts of (MHD) flow on of non-Newtonian as CNTs nanofluid were investigated by Khan *et al.* (2014), and Aman *et al.* (2017). Several other significant kinds of researches on CNTs based nanofluid have been checked by Khalid *et al.* (2018); Sallam and Hwang (1984), Nasir *et al.* (2018), Ding *et al.* (2006), and Alsagri *et al.* (2019)

Recently, MHD free convection have had great solicitation and importance because of the high wide modern industrial implementations, and thus many articles have been published in this

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field. MHD free convection with aligned magnetic radiation and variable viscosity on a semi-infinite flat plate were investigated by Seddeek (2002), and Makinde (2012) who considered the collective impacts of Navier slip and Newtonian heat on an unsteady (MHD) magnetic boundary layer flow over a flat plate. The effects of magneto-hydrodynamics (MHD) heat transfer and entropy generation flow with the influence of slip-on the flat plate were studied by Ellahi *et al.* (2018). The (MHD) flow with suitable nanoparticles as Ag suspended in water-based nanofluid over a flat plate in a porous medium, and unsteady magneto-hydrodynamic free convection flow on an infinite vertical flat plate under the characteristics of the uniform transverse magnetic field were considered by Upreti *et al.* (2018), and Palani and Srikanth (2009). Another studies concerned with the MHD flow on a solid sphere and a horizontal circular cylinder, are Qadan *et al.* (2019), Alkaskasbeh *et al.* (2014), Subba Rao *et al.* (2017) Alkaskasbeh (2018), Prasad *et al.* (2012), Alwawi *et al.* (2020); and Sheikholeslami *et al.* (2015).

The present study has discussed the Heat and mass transfer effect of free convection on MHD Casson nanofluid over a stretching sheet subject to the constant wall temperature. At the same time, it has also used the similar transformations to transform the governing equations in the dimensional form to PDEs. The reduced PDEs are solved numerically by using Keller-box scheme. The numerical calculations for various physical parameters are carried out up to a required level of accuracy (10^{-5}). The results of velocity and temperature profiles are plotted through graphs. The results of the skin friction and Nusselt number are calculated carefully and showed in a form of table.

2. MATHEMATICAL FORMULATION

Consider the free convection two-dimensional boundary layer flow of incompressible magnetohydrodynamic (MHD), Casson nanofluid over stretching sheet, which the flow begun at $y = 0$, and being restricted in $y > 0$, as presented in fig. 1, where u_w , u_s are shrinking/stretching velocity and straining velocity, respectively. Two types of Carbon Nanotubes (CNTs) nanoparticles, such as single walled carbon nanotubes (SWCNTs) and multi walled carbon nanotubes (MWCNTs), in two based fluids namely water and human blood, are investigated. The nanofluid is permeated in a Casson medium and a variable magnetic field strength B_0 and applied in the cross direction of the fluid flow. The flow is supposed to be in the x -direction, which is taken along the stretching sheet in the upward direction and the y -axis is perpendicular to it. Accordingly the same initial temperature is preserved. Instantaneously they are increased to a prescribed wall temperature $T_w > T_\infty$ the ambient temperature of the fluid that still unchanged. The impacts of nanoparticle volume fraction, Casson, magnetic, and electrical conductivity have merged in the momentum equation.

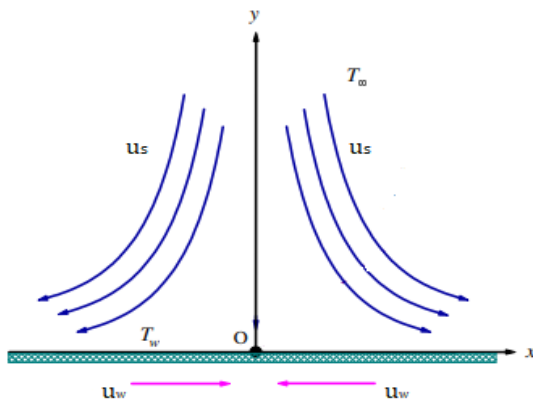


Fig. 1 Physical sketch of the problem.

The equation obtained below for determining the rheological property of Casson fluid. Casson (1959)

$$\tau_{ij} = \begin{cases} 2(\mu_B + p_y/\sqrt{2\pi})e_{ij} & \pi > \pi_c, \\ 2(\mu_B + p_y/\sqrt{2\pi_c})e_{ij} & \pi < \pi_c, \end{cases} \quad (1)$$

Here, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j) -th component of the deformation rate, π_c , p_y , and μ_B respectively are critical values of this product based on the non-Newtonian model, the yield stress of the fluid, and the plastic dynamic viscosity of the non-Newtonian fluid.

In the case of the Casson fluid, $\pi > \pi_c$ so:

$$\mu = \mu_B + p_y/\sqrt{2\pi} \quad (2)$$

Substitute the value of $p_y = \mu_B\sqrt{2\pi}/\beta$ in equation (2), then the kinematic viscosity of the Casson fluid, which μ_B is purely dependent on plastic dynamic viscosity, ρ is density and β is parameter of the Casson fluid as

$$\frac{\mu}{\rho} = \frac{\mu_B}{\rho} + \left(1 + \frac{1}{\beta}\right) \quad (3)$$

Depending on the previous assumptions, the governing equations, the continuity, momentum and energy equations, for Casson nanofluid over a stretching sheet can be presented as follows:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \quad (4)$$

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = \nu_{nf} \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} - \frac{\sigma_{nf}}{\rho_{nf}} B_0^2 \tilde{u}, \quad (5)$$

$$\tilde{u} \frac{\partial T}{\partial \tilde{x}} + \tilde{v} \frac{\partial T}{\partial \tilde{y}} = \frac{K_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial \tilde{y}^2} \quad (6)$$

Subject to the constant wall temperature boundary condition:

$$\begin{aligned} \tilde{u} = u_w(\tilde{x}) = a\tilde{x}, \tilde{v} = 0, T = T_w \text{ at } \tilde{y} = 0, \\ \tilde{u} \rightarrow 0, T \rightarrow T_\infty \text{ as } \tilde{y} \rightarrow \infty, \end{aligned} \quad (7)$$

where \tilde{u} and \tilde{v} are the velocity components along the \tilde{x} and \tilde{y} directions, respectively. The properties of nanofluid are expressed by Ahmed and Khan (2018), and Swalmeh *et al.* (2019 a) as follows:

$$\begin{aligned} \rho_{nf} &= (1 - \chi)\rho_f + \chi\rho_s, \mu_{nf} = \frac{\mu_f}{(1 - \chi)^{2.5}}, \\ (\rho c_p)_{nf} &= (1 - \chi)(\rho c_p)_f + \chi(\rho c_p)_s, \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_s + 2k_f) + \chi(k_f - k_s)}, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \\ \sigma_{nf} &= \left[1 + \frac{3((\sigma_s/\sigma_f) - 1)\chi}{((\sigma_s/\sigma_f) + 2) - ((\sigma_s/\sigma_f) - 1)\chi} \right] \sigma_f. \end{aligned} \quad (8)$$

Using the following similarity transformations in the governing equations (4) to (6) subject to the boundary conditions (7) Salleh *et al.* (2010):

$$\begin{aligned} \tilde{u} = \frac{\partial \psi}{\partial \tilde{y}}, \tilde{v} = -\frac{\partial \psi}{\partial \tilde{x}} \\ \psi = (a\tilde{v})^{1/2} \tilde{x}f(\eta), \quad \eta = (a\tilde{v})^{1/2} \tilde{y}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \end{aligned} \quad (9)$$

where ψ is the stream function .

By substituting the equations (6) and (7) we get the following equations

$$\left(\frac{\rho_f}{\rho_{nf}} \frac{1}{(1 - \chi)^{2.5}} \right) \left(1 + \frac{1}{\beta} \right) f''' + ff'' - (f')^2 - \frac{\rho_f}{\rho_{nf}} \frac{\sigma_{nf}}{\sigma_f} Mf' = 0 \quad (10)$$

$$\frac{1}{Pr} \left[\frac{k_{nf}/k_f}{(1-\chi) + \chi(\rho c_p)_s / (\rho c_p)_f} \right] \theta'' + f\theta' = 0, \quad (11)$$

and the corresponding boundary condition are

$$f' = 1, f = 0, \theta = 1 \text{ as } \eta = 0, \\ f' \rightarrow 0, \theta \rightarrow 0, \text{ as } \eta = \infty. \quad (12)$$

Here we have $Pr = \frac{\nu_f}{\alpha_f}$ is called the Prandtl number and

$$M = \left(\frac{\sigma_f B_0^2 a^2}{\rho_f \nu_f} \right) \text{ is magnetic parameter.}$$

For the engineering usefulness, the physical quantities especially, the local skin friction coefficient (C_f) and the Nusselt number (Nu), the model are followed respectively,

$$C_f = \left(\frac{\tau_w}{\rho U_w^2} \right)_{\bar{y}=0}, \text{ and } Nu = \left(\frac{a q_w}{k_f (T_w - T_\infty)} \right)_{\bar{y}=0}, \quad (13)$$

with τ_w and q_w are corresponding, shear stress, and heat flux on the plane of the wall are defined below.

$$\tau_w = \mu_{nf} \left(\frac{\partial^2 u}{\partial \bar{y}^2} \right)_{\bar{y}=0} \text{ and } q_w = -k_{nf} \left(\frac{\partial T}{\partial \bar{y}} \right)_{\bar{y}=0}. \quad (14)$$

Now, the local skin friction coefficient and Nusselt number become as:

$$Re^{1/2} C_f = \frac{1}{(1-\chi)^{2.5}} \left(1 + \frac{1}{\beta} \right) f''(0), \quad Re^{-1/2} Nu = -\frac{k_{nf}}{k_f} \theta'(0), \quad (15)$$

where $Re^{1/2} = (a \bar{x}^2 / \nu_f)$ is denoted the local Reynolds number.

3. RESULTS AND DISCUSSION

The numerical solutions obtained by using the Keller box method of the above governing equations (10) and (11) subject to the boundary condition (12) are presented in figures and tables, see Keller and Bramble (1970) and Cebeci and Bradshaw (2012). Many convection boundary layer problems use this method to study the numerical solutions and its physical quantities behaviors, this method appears to be the most malleable of the used methods and in spite of novel expansions in other numerical methods, stills a potent and very accurate process for parabolic boundary layer flows. It is also being easily adjustable to solve equations of any order and unconditionally stable on the solutions. The Keller box method procedures are: firstly, reduce PDEs to a first-order system by using a finite difference scheme. Next, Write the difference equations by using central differences, and linearize the resulting differences equations by Newton's method and write them in the matrix-vector form. Finally, solve the linear system by the block tri-diagonal elimination technique, and then writing the steps of Keller-box method in Matlab program see Swalmeh *et al* (2019 a).

Table 1: Different values of thermophysical properties of CNTs of two base Casson nanofluids Alsagri *et al.* (2019)

Physical properties	Based fluids		CNTs nanoparticles	
	Water	Human blood	SWCNT	MWCNT
k (W/mK)	0.613	0.492	6600	3000
ρ (kg/m ³)	997.1	1053	2600	1600
c_p (J/kgK)	4179	3594	425	796
δ s(Sm ⁻¹)	5.5×10^{-6}	0.8	10^6 - 10^7	1.9^{-4}
Pr	6.2	24		

Two kinds of Carbon Nanotubes (CNTs) nanoparticles, namely single and multi-walled carbon nanotubes (SWCNTs), (MWCNTs), in two based fluids namely, water and human blood. The impact of various values of nanoparticle volume fraction $0 \leq \chi \leq 0.2$, Casson $\beta \geq 1$, and magnetic parameters $M \geq 0$ on the temperature $\theta(0)$ and velocity $f'(0)$ profiles have been outlined in figures (2-7). Meanwhile, the Nusselt number and local skin friction coefficient have been enumerated and put up in the table 3. Thermo-physical characteristics of based fluids and nanoparticles are set in table 1. Also, table 2 shows the comparison values with the previous literature stated by Hassanien *et al.* (1998) and Salleh *et al.* (2010). It is noted that the results of the present study are in a good agreement.

Table 2: Comparison of $Re^{-1/2} Nu$ with viscous Newtonian fluid ($M = 0$, $\beta = \infty$, and $\chi = 0$), with several values of Pr.

Pr	$Re^{-1/2} Nu$		
	Hassanien <i>et al.</i> (1998)	Salleh <i>et al.</i> (2010)	Present
0.72	0.46325	0.46317	0.46316
1	0.58198	0.58198	0.58198
3	1.16525	1.16522	1.16524
5		1.56806	1.56807
7		1.89548	1.89550
10	2.30801	2.30821	2.30820
100	7.74925	7.76249	7.76250

Table 3: Values of the skin friction coefficient $Re^{1/2} C_f$ and local Nusselt number $Re^{-1/2} Nu$ for a different values of Casson parameter β , χ and M .

(a)

β	χ	M	SWCENT Human blood		MWCENT Human Blood	
			$Re^{1/2} C_f$	$Re^{-1/2} Nu$	$Re^{1/2} C_f$	$Re^{-1/2} Nu$
3	0.075	2	-7.7591	5.1829	-7.7213	5.1613
7			-16.7940	5.1472	-16.682	5.1267
10			-23.5388	5.1381	-23.384	5.1176
3	0.1	5	-11.9797	5.6423	-11.818	5.6152
	0.15		-14.2650	7.1565	-14.027	7.0985
	0.2		-17.0390	9.0564	-16.734	8.9481
3	0.15	1	-8.2849	7.4889	-8.2742	7.4182
		2	-10.1148	7.3874	-10.027	7.3211
		3	-11.6647	7.3014	-11.516	7.2384
		7	-16.4589	7.0339	-16.152	6.9798

(b)

β	χ	M	SWCENT Water		MWCENT Water	
			$Re^{1/2} C_f$	$Re^{-1/2} Nu$	$Re^{1/2} C_f$	$Re^{-1/2} Nu$
3	0.075	2	-7.8666	2.4157	-8.6175	2.3634
7			-16.997	2.3761	-18.616	2.3247
10			-23.823	2.3712	-26.092	2.3146
3	0.1	5	-12.224	2.5265	-14.15	2.4049
	0.15		-14.708	3.2264	-18.384	2.9946
	0.2		-17.755	4.1124	-24.026	3.7209
3	0.15	1	-8.4326	3.5758	-9.8259	3.4665
		2	-10.364	3.4680	-12.526	3.3159
		3	-11.988	3.3774	-14.740	3.1931
		7	-16.998	3.1008	-21.418	2.8347

The impact of β , χ , and M on physical quantities, such as

$Re^{-1/2} Nu$ and $Re^{1/2} C_f$ have been summarized in table 3. In this table, we perceived that an increase in the three parameters, β , χ , and M , leads to minimizing the local skin friction coefficient in the existence of CNTs nanoparticles suspended in water and human blood based fluids. Besides, as an increase in the β and M , it leads to a decrease in $Re^{-1/2} Nu$ of the CNTs nanoparticles based water and human blood. But the opposite case happens, when the χ is increased, then leads to an increment in $Re^{-1/2} Nu$ and $Re^{1/2} C_f$. Besides, it has been observed from the table that CNTs/Human blood has a high $Re^{-1/2} Nu$ and $Re^{1/2} C_f$ compared with CNTs/water Casson nanofluid. In addition, SWCNT has lower $Re^{1/2} C_f$ than MWCNT based Human blood Casson nanofluid, but the opposite case happens in water-based Casson nanofluid. Also, SWCNT human blood/water has high $Re^{-1/2} Nu$ compared with MWCNT human blood/water.

thickness, but the opposite case happens in MWCENT and SWCNT that are suspending in water.

Figures 4 and 5 exhibit that when the M increase, then the temperature increase, while the velocity profiles are decrement. Practically it has been noticed that the presence of a magnetic field in the flow field region decelerates the fluid motion. The resistive force created is known as the Lorentz force. So, the Lorentz force slows down the velocity of the fluid. In addition, human blood has high temperatures and low velocity compared to water with an increase in the values of the M . Moreover, MWCENT has a high velocity and temperature than SWCNT suspended in human blood Casson nanofluid, meanwhile, when the CNTs in water, we noticed that the SWCNT has a lower temperature than MWCENT nanoparticles, but the reverse case happens in the velocity profile.

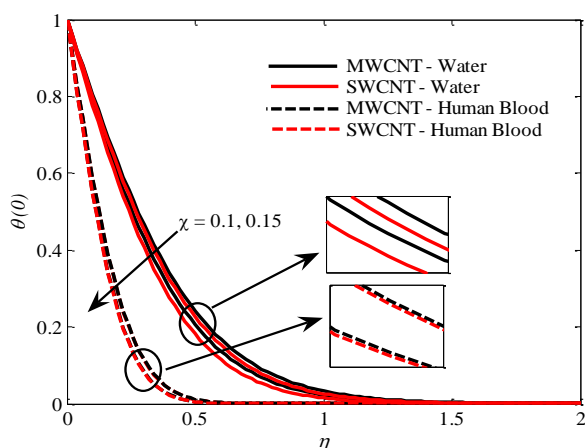


Fig. 2 Temperature profile vs χ .

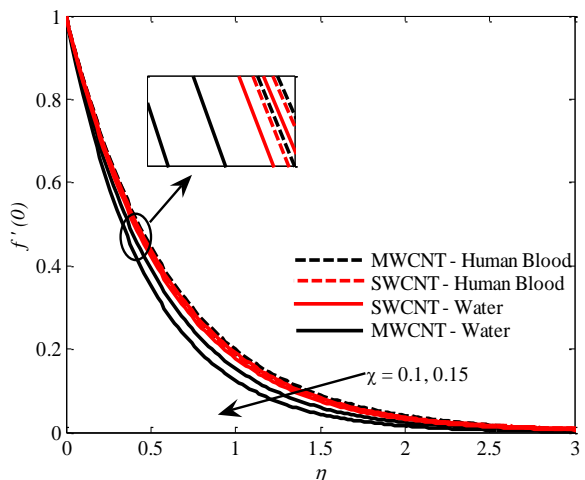


Fig. 3 Velocity profile vs χ .

Figures 2 and 3 show that both results of the quantities of the temperature and velocity are reduced with an increment of values of the χ . Moreover, it is noted that CNTs/human blood has a lower temperature than CNTs/water, but the reverse case happens in the velocity profile with an increase in the value of thermal boundary layer thickness. Also, the MWCENT nanoparticle has a higher temperature opposite to SWCNT nanoparticles, and the reason is that the density of SWCNT is greater than it of MWCNT. Moreover, it can be seen that SWCNT/human blood has a lower velocity than MWCENT/human blood with an increment of the value of thermal boundary layer

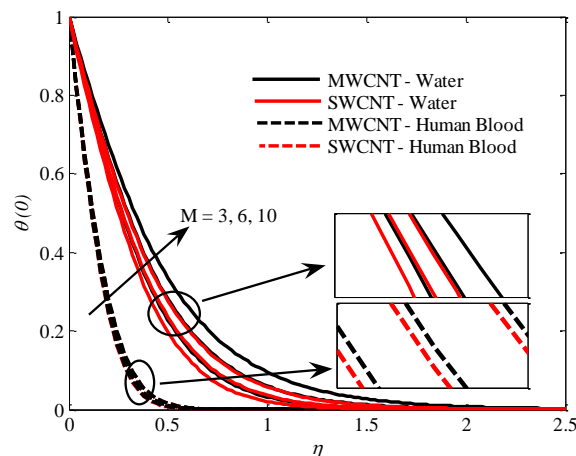


Fig. 4 Temperature profile vs M .

Figures 6 and 7 display the effects of the β on the temperature and velocity profiles of Casson nanofluid. It is observed that the increasing in β values, leads to increasing in the temperature and decreasing in the velocity profiles, in the presence of CNTs suspended in human blood/water Casson nanofluid. Furthermore, it can be clearly seen that the MWCENT has high temperature than SWCNT nanoparticles. Besides, the MWCENT has less velocity than SWCNT in water, but the opposite case happens when the CNTs nanoparticles suspended in human blood based Casson nanofluid, the MWCENT has high velocity than SWCNT in human blood. On the other hand, human blood has high velocity and less temperature compared with water.

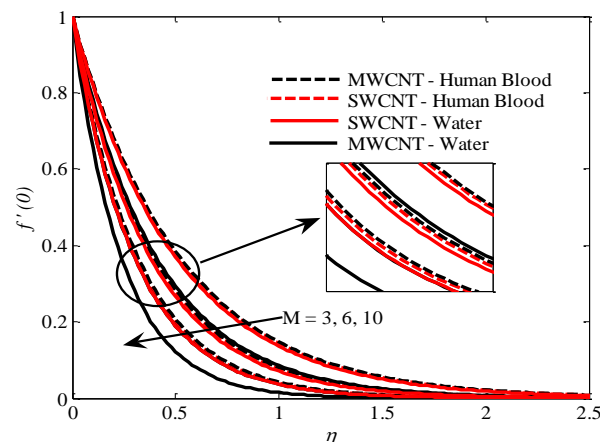


Fig. 5 Velocity profile vs M .

human blood/water has a high local Nusselt number compared with MWCENT human blood/water.

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NOMENCLATURE

B	Thermal expansion coefficient
C_f	Local skin friction coefficient
c_p	Specific heat
g	Acceleration due to gravity
Gr	Grashof number
K	Material or micropolar parameter
k	Thermal conductivity
M	Magnetic parameter
Nu	Local Nusselt number
Pr	Prandtl number
p_y	Yield stress of the fluid
q_w	Constant wall heat flux
T	Fluid temperature
T_∞	Temperature of the ambient fluids
T_w	Wall temperature
\tilde{u}, \tilde{v}	Non-dimensional velocity components along \tilde{x} and \tilde{y} directions
\tilde{x}, \tilde{y}	Coordinates measured from the lower stagnation point along the surface of cylinder and Normal to it, respectively

GREEK SYMBOLS

α	Thermal diffusivity
β	Parameter of the Casson fluid
θ	Non-dimensional temperature
μ	Dynamic viscosity
μ_B	Plastic dynamic viscosity of the non-Newtonian fluid
π_c	Critical value of this product based on the non-Newtonian model
κ	Vortex viscosity
σ_f	Electric conductivity
α	Thermal diffusivity
ρ	Fluid density
χ	Volume fraction
ψ	stream function

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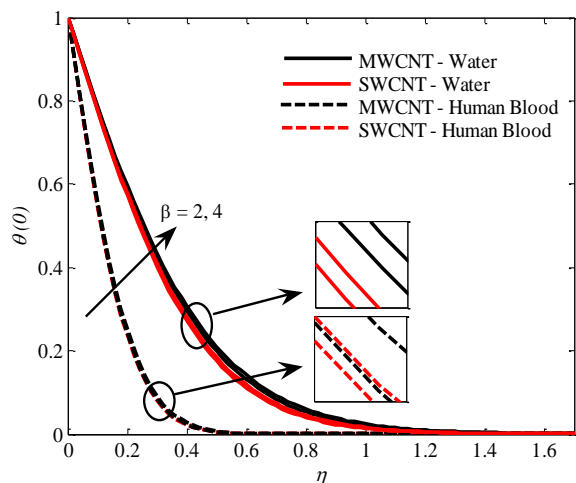


Fig. 6 Temperature profile vs β .

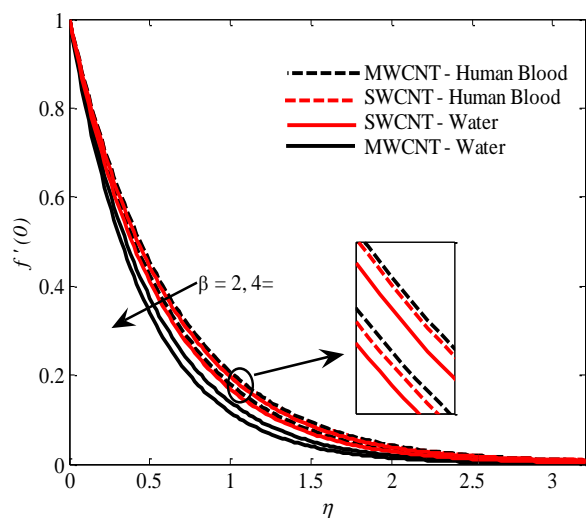


Fig. 7 Velocity profile vs β .

4. CONCLUSIONS

This paper offers a facilitated mathematical model and numerically studies the heat transfer effects for free convection in MHD Casson nanofluid flow over a stretching sheet. Several properties of the MHD Casson nanofluid parameters on the physical quantities, such as the local skin friction coefficient and the local Nusselt number, as well as temperature and velocity are considered in an attempt to analyse them through various plots and tables. In addition the constant wall temperature boundary condition has been considered in this study. The following results are the findings of the study:

1. The temperature profile of water is more than it for human blood based Casson nanofluid when the values of Casson, magnetic, and nanoparticle volume fraction parameters are increased.
2. When MHD Casson nanofluid parameters increase, local skin friction coefficient in SWCNT/human blood is less than it in MWCENT/human blood based Casson nanofluid.
3. MWCENT human blood /water has a higher temperature profile compared with SWCENT human blood /water with an increase in the MHD Casson nanofluid parameters.
4. SWCENT has lower local skin friction than MWCENT based Human blood Casson nanofluid, but the opposite case happens in water-based Casson nanofluid. Also, SWCENT

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