



NUMERICAL AND EXPERIMENTAL INVESTIGATIONS OF NATURAL CONVECTIVE HEAT TRANSFER FROM TWO-SIDED DIAGONALLY INCLINED SQUARE PLATES HAVING A FINITE THICKNESS

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ABSTRACT

Natural convective heat transfer from two-sided diagonally inclined square plates having various thicknesses has been numerically and experimentally investigated. The aim of this work is to determine the influence of the plate thickness and diagonal inclination angle on the heat transfer rate for various flow regimes. The mean heat transfer rate was numerically obtained using ANSYS FLUENT© and experimentally determined using the Lumped Capacity Method. The results indicate that the plate thickness does not have a significant influence on the heat transfer rate while the diagonal inclination angle significantly influences the heat transfer rate especially at higher Rayleigh number values considered.

Keywords: *Natural Convection, Thick Plate, Diagonally Inclined, Numerical, Experimental*

1. INTRODUCTION

A numerical study of natural convective heat transfer from the top, bottom and side surfaces of two-sided diagonally inclined square plates with various dimensionless thicknesses (thickness-to-side length ratios) and for various diagonal inclination angles have been conducted for a wide range of Rayleigh number. A limited range of experiments has been undertaken for validation purposes. The upper, lower and side surfaces of the plate are at higher temperature than that of the surrounding fluid. The situation considered is shown in Fig. 1. Mean heat transfer rates from the upper surface, from the lower surface and from the side surface of the plate as well as the mean heat transfer rate averaged over the heated surfaces of the plate have been considered.

Natural convective heat transfer from plates where all surfaces are heated to the same temperature appear in a variety of engineering applications, for example, in cooling of electrical and electronic components. In such applications the heated plate may in some cases be diagonally inclined to the horizontal with a certain inclination angle. The situation considered in the present study is a simplified model of those arising in such engineering applications. However, the results obtained in this study should give an indication of the effects of the diagonal angle of inclination and of the plate thickness in practical situations.

A limited number of numerical and experimental studies of natural convective heat transfer from inclined plates is available, see (Hassan and Mohamed, 1970; Pera and Gebhart, 1973; Al-Arabi and Sakr, 1988; Kobus and Wedekind, 2002; Wei *et al.*, 2002; Corcione *et al.*, 2011; Oosthuizen, 2014). All studies undertaken used air as the surrounding fluid. In these studies, however, only simple forms of inclination have been considered, i.e., situations in which the angle of inclination is with respect to a side of the plate. Furthermore, most of these studies have only considered situations with a limited range of flow conditions, i.e., have only considered laminar flow and situations where only one surface of

the plate is heated, i.e., a single-sided plate. An early study of the natural convective heat transfer from the inclined flat surfaces was undertaken by Hassan and Mohamed (1970) who carried out experiments to measure the local heat transfer coefficients along an isothermal flat plate for a laminar flow regime. The plate was single-sided and the range of inclination angles considered was from horizontal (plate is facing upward) to the horizontal (plate is facing downward) positions. Local Nusselt number variations were correlated in terms of the local Grashof number for various values of the inclination angle. Pera and Gebhart (1973) later analytically and experimentally investigated the natural convective heat transfer above horizontal and slightly inclined surfaces with laminar flow condition being considered. Surface thermal boundary conditions of uniform temperature and uniform heat flux were considered. The similarity solution approach was used to describe the temperature and velocity distributions and the coupled governing equations were integrated numerically. Correlations describing the local Nusselt numbers for Prandtl numbers near unity were presented for both surface thermal boundary conditions. In addition, the effect of inclination angle on local Nusselt number was shown and compared with the theoretical values for Prandtl number of 0.7. Al-Arabi and Sakr (1988) obtained experimental local and average heat transfer data for natural convective heat transfer from isothermal vertical and inclined single-sided upward-facing plates in both the laminar and the turbulent regions. The range of inclination angles considered was from 0° to 80° measured from the vertical. Equations representing the heat transfer results of this study were proposed. More recently, Kobus and Wedekind (2002) performed experiments measuring the natural convective heat transfer from two-sided thin circular disks of different thickness-to-diameter ratio at arbitrary angles of inclination between vertical and horizontal. The authors used their experimental data to develop an empirical correlation for a range of Rayleigh numbers from 2×10^2 to 3×10^7 . Wei *et al.* (2002) undertook a numerical study of the natural convective heat transfer from the upper and lower surfaces of uniformly heated thin plates with various

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thickness-to-width ratios at arbitrary angles of inclination from horizontal (angles of 0° to 90° being considered). The range of Raleigh numbers considered was from 4.8×10^6 to 1.87×10^8 . It was concluded that for an inclination angle of less than 10°, the heat transfer characteristics of the upper and lower surfaces are different. Therefore, the average Nusselt number for the upper and lower surfaces were expressed in terms of two different equations, whereas for inclination angle greater than or equal to 10°, the average Nusselt number for the upper and lower surfaces could be adequately correlated by a single equation. Corcione *et al.* (2011) numerically studied the laminar natural convective heat transfer from an inclined two-sided plate immersed in different fluids and whose sides are simultaneously heated to the same uniform temperature and the situation where one surface was isothermal while the other surface was adiabatic (single-sided case). The range of inclination angles considered was from 0° to 75° measured from the vertical, the range of the Rayleigh numbers was from 10 to 10^7 and the range of the Prandtl numbers was from 0.7 to 140. Correlation for the average Nusselt numbers were developed. The average Nusselt numbers for the vertical orientation at different values of Rayleigh and Prandtl numbers were compared with the theoretical solution results given by Churchill and Chu (1975) for a single sided plate and by Hassan and Mohamed (1970) for a two-sided plate with good agreement. Oosthuizen (2014) numerically investigated the natural convective heat transfer from an inclined isothermal single-sided upward-facing square flat plate embedded in a flat adiabatic surrounding surface. The range of inclination angles considered was from 0° to 30° measured from the horizontal and the range of Rayleigh numbers was from 10^5 to 10^{13} . The *k*-epsilon turbulence model was used to obtain the solution and the results showing the variation of the Nusselt numbers with the inclination angles were obtained.

The literature review has demonstrated that there are no available studies to predict the influence of the plate thickness and the diagonal inclination angle on the heat transfer rate from the surfaces of a square plate when it is diagonally inclined to the horizontal. This is basically the goal of the present research.

A summary of this study was presented at the 14th International Conference of Heat Transfer, Fluid Mechanics and Thermodynamics (HEFAT), Wicklow, Ireland, July 22-24, 2019, see (Manna and Oosthuizen, 2019).

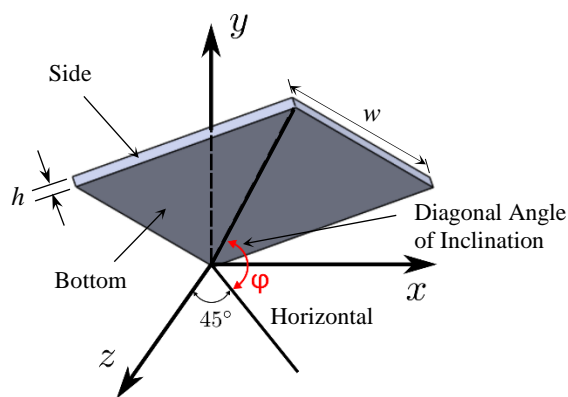


Fig. 1 Finite thickness square plate situation considered. Also shown are the definitions of the plate thickness, *h*, of the bottom (lower) surface and of the side surface of a two-sided plate. The top (upper) surface is above the bottom surface.

2. NUMERICAL SOLUTION PROCEDURE

The flow has been assumed to be steady and three dimensional and the fluid properties have been assumed constant except for the density change with temperature which gives the rise to the buoyancy forces, i.e., the Boussinesq approximation has been adopted. The standard *k*-epsilon turbulence model with full account being taken to the buoyancy force effects has been used. This turbulence model has been applied in all

calculations and is thus used to determine when transition begins. This approach has been widely used in previous studies, e.g., see (Albets-Chico *et al.*, 2008; Kalendar *et al.*, 2016; Plumb and Kennedy, 1977; Savill, 1993; Schmidt and Patankar, 1991; Xamán *et al.*, 2005). These studies indicate that the results obtained using this approach are of acceptable accuracy for the type of flow situation being considered in the present study. The solution has been obtained by numerically solving the standard continuity, Navier-Stokes, and energy equations using the commercial CFD solver ANSYS FLUENT®. An extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken, and the heat transfer results presented here are grid and convergence-criteria independent to less than one per cent. Some typical results obtained here in the grid-independence testing are shown in Table 1. These results are for the case of natural convective heat transfer from diagonally inclined square plate which has a dimensionless thickness, *H* of zero and at a Rayleigh number of 10^7 for diagonal inclination angles of 0° and 30°. The number of nodes used was 2,610,000.

Table 1 Results for the mesh independence study for $H=0$, $Ra=10^7$ and $\phi=0^\circ$ and 30° .

Angle (degrees)	Rayleigh number	Number of nodes	Nu _{bot}	Nu _{top}	Nu _{total}
0	1.E+07	1957500	50.6490	43.5737	47.1113
		2272500	50.3053	41.1845	45.7449
		2610000	48.8119	40.0796	44.4457
		2969600	48.8741	39.8956	44.3849
30	1.E+07	1957500	52.4275	42.1494	47.2885
		2272500	51.6781	40.5360	46.1070
		2610000	50.3174	39.4418	44.8796
		2969600	50.3844	39.3217	44.8530

3. NUMERICAL RESULTS AND DISCUSSION

The solution has the following parameters:

1. The Rayleigh number, *Ra*, based on the side length, *w*, of the square plate and the difference between the plate temperature, *T_w*, and the temperature of the fluid far from the plate, *T_f*, i.e.:

$$Ra = \frac{\beta g w^3 (T_w - T_f)}{\nu \alpha} \quad (1)$$

2. Diagonal inclination angle, ϕ .
3. Plate dimensionless thickness, *H*, defined by:

$$H = h/w \quad (2)$$

4. The Prandtl number, *Pr*.

Results have been obtained only for a Prandtl number of 0.74, which is effectively the value for air. The reason for this is that much of the equipment that is cooled by natural convection uses air as the heat transfer fluid. Diagonal inclination angles of between 0° and 40° and Rayleigh numbers between approximately 10^4 and 10^{14} have been considered.

The mean heat transfer rate from the top surface, \bar{Q}_{top} , from the bottom surface, \bar{Q}_{bot} , and from the side surface, \bar{Q}_{side} , of the plate as well as the mean heat transfer rate averaged over all heated surfaces, \bar{Q} , have been expressed in terms of a mean Nusselt number based on the side length of the square plate and the difference between the plate temperature and the temperature of the fluid far from the plate.

The following Nusselt numbers have, therefore, been introduced:

$$Nu_{total} = \frac{\bar{Q}w}{A_{total}(T_w - T_f)k} \quad (3)$$

$$Nu_{top} = \frac{\bar{Q}_{top}w}{A_{plate}(T_w - T_f)k} \quad (4)$$

$$Nu_{bot} = \frac{\bar{Q}_{bot}w}{A_{plate}(T_w - T_f)k} \quad (5)$$

$$Nu_{side} = \frac{\bar{Q}_{side}w}{A_{side}(T_w - T_f)k} \quad (6)$$

where,

$$\bar{Q} = \bar{Q}_{top} + \bar{Q}_{bot} + \bar{Q}_{side} \quad (7)$$

and where A_{total} , A_{plate} , and A_{side} are the total area of the heated surfaces of the plate, the areas of the upper and the lower surfaces of the plate, and the area of the side surface of the plate, respectively.

Therefore:

$$A_{total} = 2A_{plate} + A_{side} \quad (8)$$

$$Nu_{total} = \frac{(Nu_{top} + Nu_{bot})A_{plate} + Nu_{side}A_{side}}{A_{total}} \quad (9)$$

Variations of the mean Nusselt number for the top surface, Nu_{top} , for the bottom surface, Nu_{bot} , and for all heated surfaces of the plate, Nu_{total} , with Rayleigh number for various diagonal inclination angles are shown in Figs. 2 and 3 for the plate dimensionless thickness values of 0.0635 and 0.3, respectively. The results given in Figs. 2-a and 3-a are for the case where the diagonal inclination angle is 0° , i.e., for the case where the plate is horizontal. It will be seen from the results given in Figs. 2 and 3 that in all cases, i.e., for all angles of inclination, the Nusselt number for the bottom surface is higher than that for the top surface at the lower Rayleigh numbers considered but that in all cases the Nusselt number for the top surface is higher than that for the bottom surface at the higher Rayleigh numbers considered.

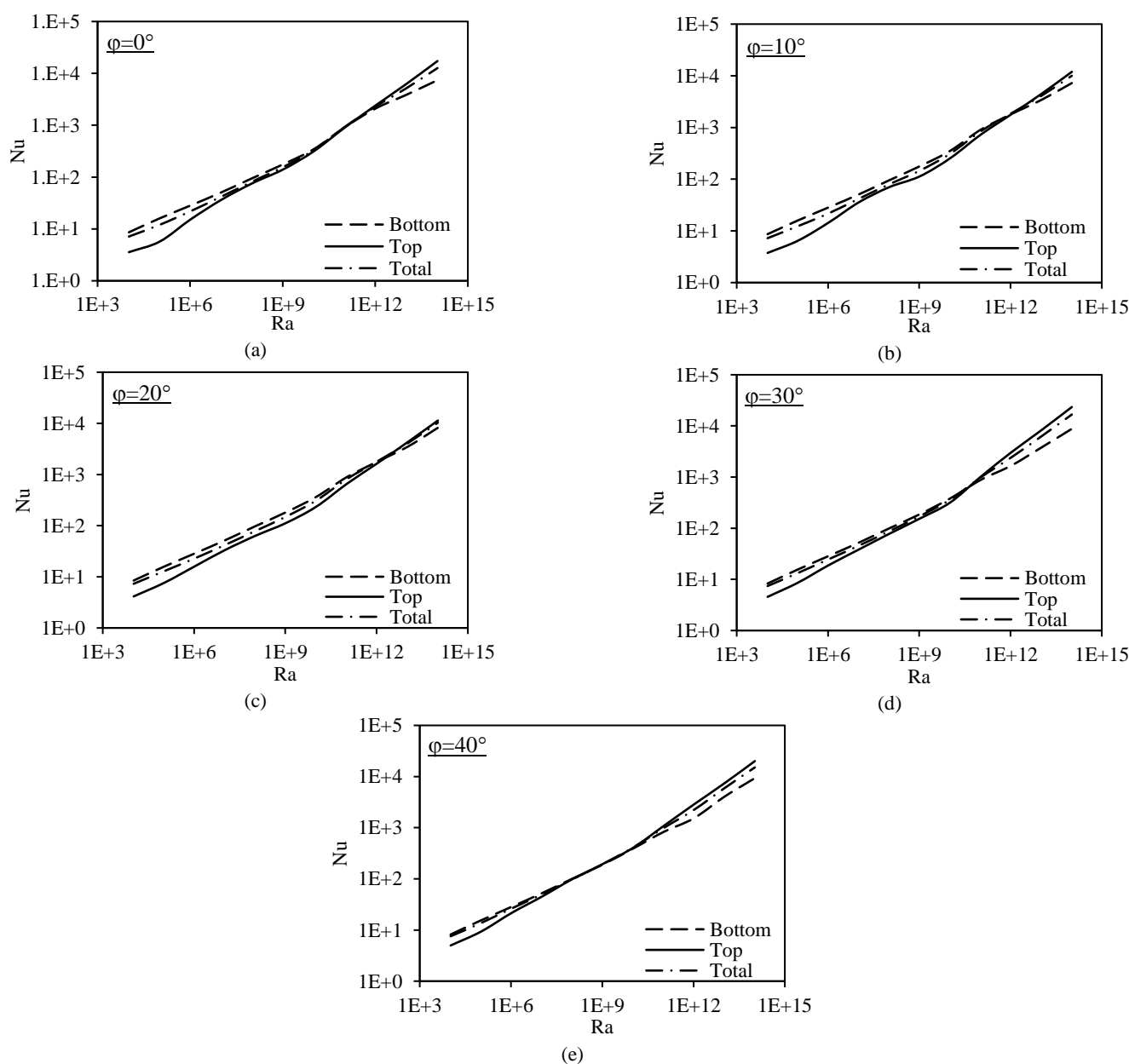


Fig. 2 Variations of the mean Nusselt number for the bottom surface, the top surface, and for all heated surfaces of the square plate with Rayleigh number for diagonal angles of inclination of (a) 0° , (b) 10° , (c) 20° , (d) 30° , and (e) 40° for a dimensionless plate thickness, H , of 0.0635.

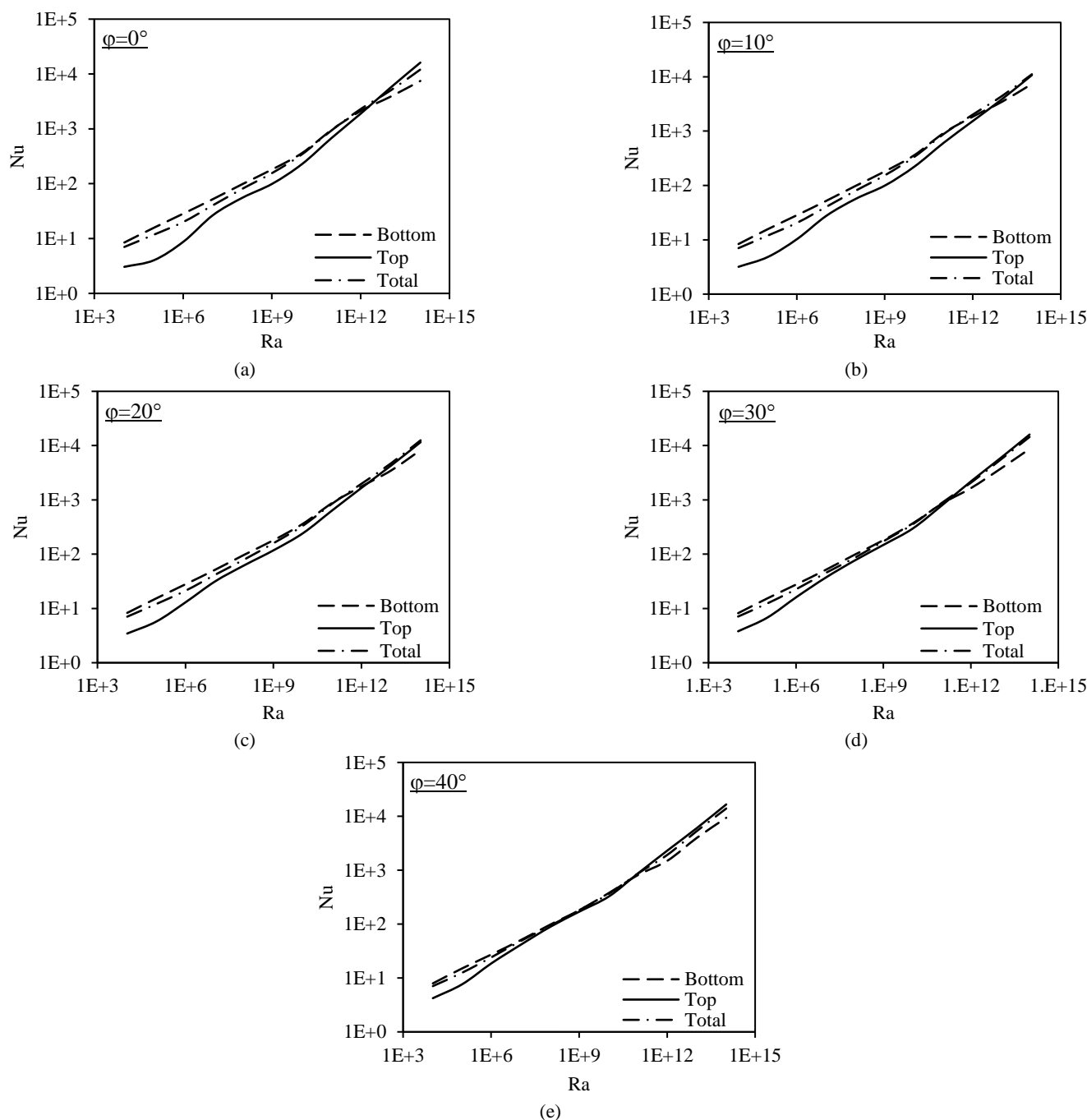


Fig. 3 Variations of the mean Nusselt number for the bottom surface, the top surface, and for all heated surfaces of the square plate with Rayleigh number for diagonal angles of inclination of (a) 0°, (b) 10°, (c) 20°, (d) 30°, and (e) 40° for a dimensionless plate thickness, H , of 0.3.

To further illustrate the effect of the plate dimensionless thickness on the heat transfer rate. Typical variations of the mean Nusselt number for the top and bottom surfaces with dimensionless plate thickness for various Rayleigh number values and for various diagonal inclination angles considered are shown in Fig. 4. It can be seen from this figure that for various diagonal inclination angles considered the dimensionless thickness has an almost negligible effect on the Nusselt number for the bottom surface whereas this effect is significant for the top surface especially at lower values of dimensionless thickness considered, the nature of this effect being dependent on the value of the Rayleigh number. It can be also seen from the figure that for the top surface the Nusselt number decreases with increasing the dimensionless thickness for various diagonal inclination angles considered. Typical variations of the mean Nusselt number with Rayleigh number for the side surface of the plate

for the diagonal inclination angles considered are shown in Fig. 5 for two values of dimensionless plate thickness. It will be seen from this figure that for the diagonal inclination angles of 0°, 10° and 20° (Fig. 5-a, b, c) the dimensionless plate thickness does affect the Nusselt number for the range of Rayleigh numbers considered except at higher values of the Rayleigh number (approximately greater than 10^{10}) where this effect is almost negligible. For the diagonal inclination angles of 30° and 40° (Fig. 5-d, e) the effect of the dimensionless plate thickness on the Nusselt number, however, is significant at lower values of the Rayleigh number (approximately less than 10^7) while this effect is negligible for Rayleigh numbers between approximately 10^7 and 10^{10} . Above Rayleigh number value of approximately 10^{10} the dimensionless plate thickness has a modest effect on the Nusselt number.

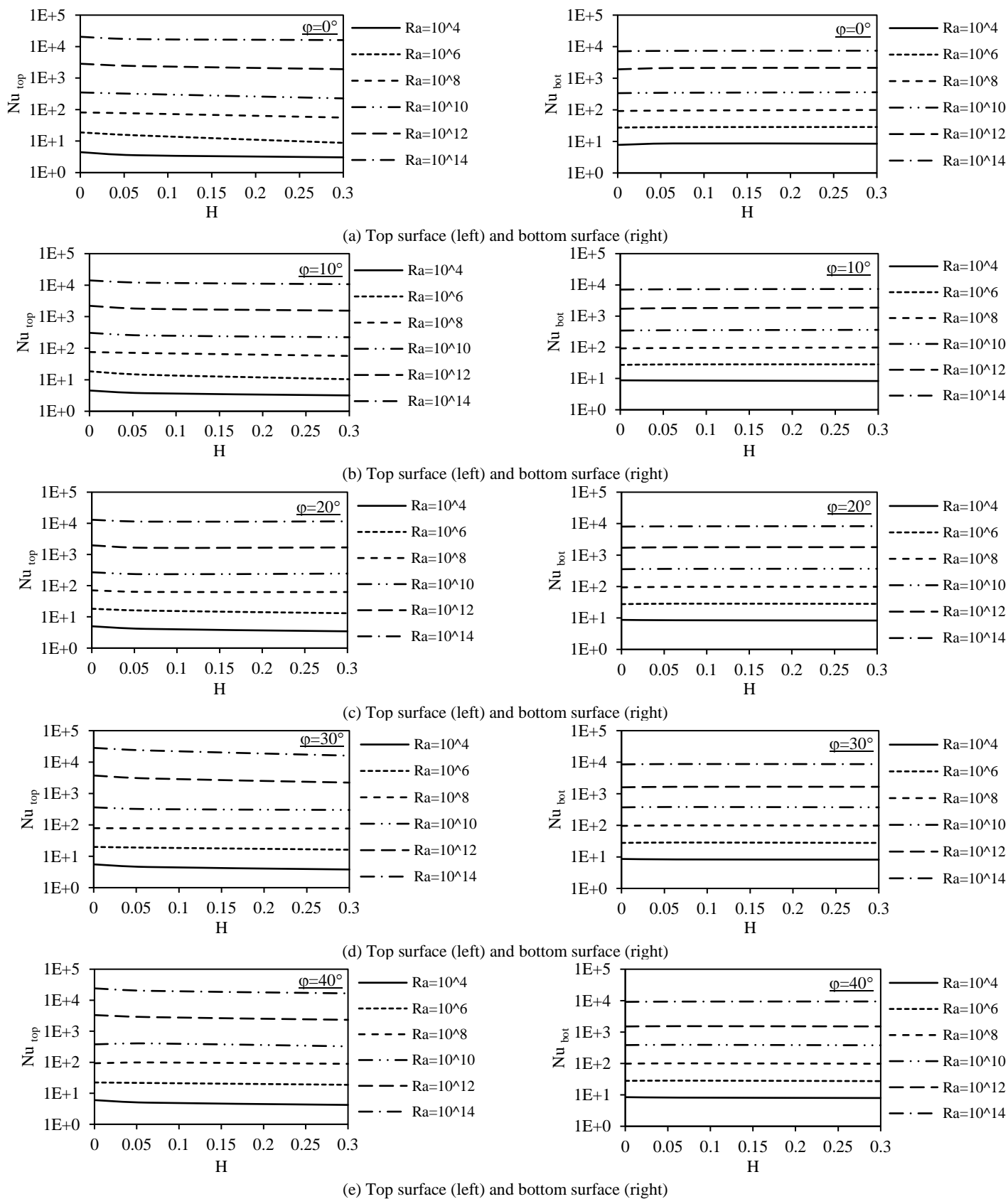


Fig. 4 Variations of the mean Nusselt number for the top and bottom surfaces of the square plate with dimensionless plate thickness for various values of Rayleigh number for diagonal inclination angles of (a) 0° , (b) 10° , (c) 20° , (d) 30° , and (e) 40° .

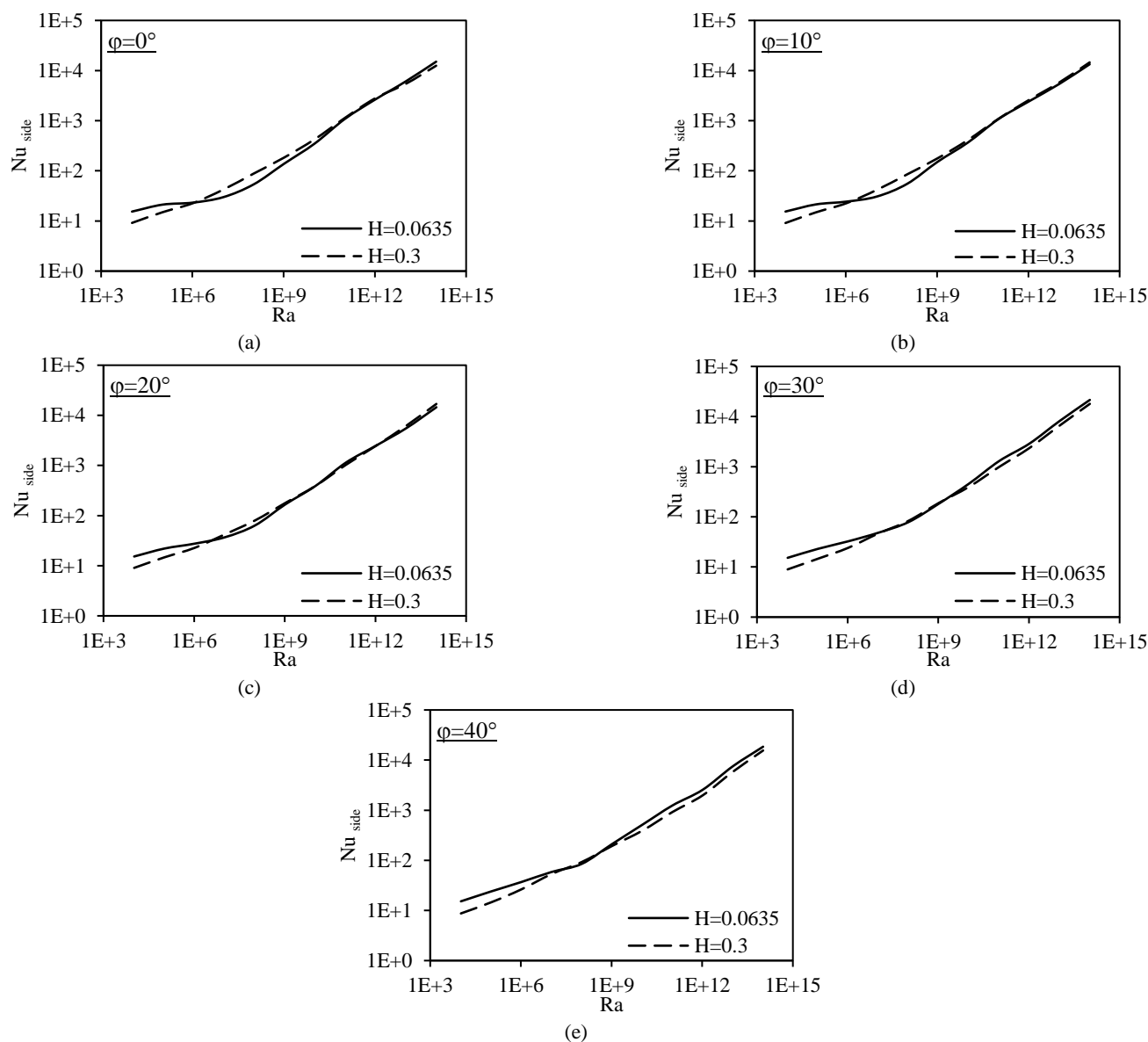


Fig. 5 Variations of the mean Nusselt number for the side surface of the plate with Rayleigh number for two values of dimensionless plate thickness for diagonal inclination angles of (a) 0° , (b) 10° , (c) 20° , (d) 30° , and (e) 40° .

To more clearly illustrate the influence of the diagonal inclination angle on the heat transfer rate, typical variations of the mean Nusselt number with diagonal inclination angle for the range of Rayleigh number considered for the dimensionless plate thickness of 0.0635 are shown in Figs. 6 and 7 for the top and bottom surfaces, respectively while typical variations for a dimensionless plate thickness of 0.3 are shown in Figs. 8 and 9 for the top and bottom surfaces, respectively. In these figures, the range of Rayleigh numbers considered was divided into three subranges, lower, intermediate and higher values of Rayleigh number. The purpose of this is to more clearly illustrate the influence of the diagonal inclination angle on the heat transfer rate. It will be seen from these figures that for the top surface of the plate which has a dimensionless plate thickness, H , of 0.0635 (Fig. 6) the Nusselt number increases as the diagonal inclination angle increases for Rayleigh number values of 10^4 and 10^5 . After Rayleigh number value of 10^5 the Nusselt number first decreases as the angle increases from 0° and starts to increase as the angle increases above about 10° for Rayleigh number value of 10^6 . For the range of Rayleigh numbers between 10^7 and 10^{11} the behavior is almost the same such that the Nusselt number decreases as the diagonal inclination angle increases from 0° but starts to increase as the angle is above about 20° . For higher values of Rayleigh number, the Nusselt number decreases as

the diagonal inclination angle increases from 0° until about 20° is reached after which the Nusselt number starts to increase with the angle and then decreases again after about 30° . For the bottom surface, it can be seen from Fig. 7 that the effect of the diagonal inclination angle on the Nusselt number is significantly different from that for the top surface such that this effect is significant only at higher values of Rayleigh number (Fig. 7-c) where the Nusselt number decreases with the diagonal inclination angle for Rayleigh number value of 10^{12} whereas for the Rayleigh number values of 10^{13} and 10^{14} the Nusselt number decreases as the diagonal inclination angle increases from 0° to about 10° after which the Nusselt number increases with the diagonal inclination angle. For the plate with a dimensionless thickness, H , of 0.3, however, it can be noted from the results shown in Fig. 8 that for the top surface the Nusselt number increases with the diagonal inclination angle for a wider range of Rayleigh number (from 10^4 to 10^9). The Nusselt number starts to decrease as the diagonal inclination angle increases from 0° and starts to increase as the angle increases above about 10° for the other values of Rayleigh number considered. The effect of the diagonal inclination angle on the Nusselt number for the bottom surface for $H=0.3$ (Fig. 9) is the same as that for $H=0.0635$ since the dimensionless thickness has an almost negligible effect on the heat transfer rate from the bottom surface.

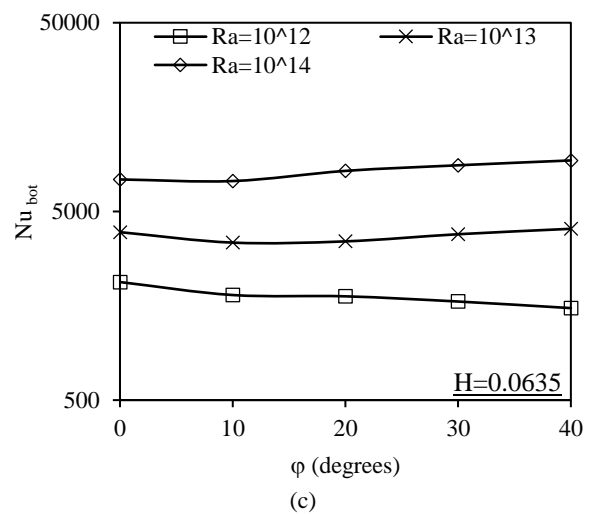
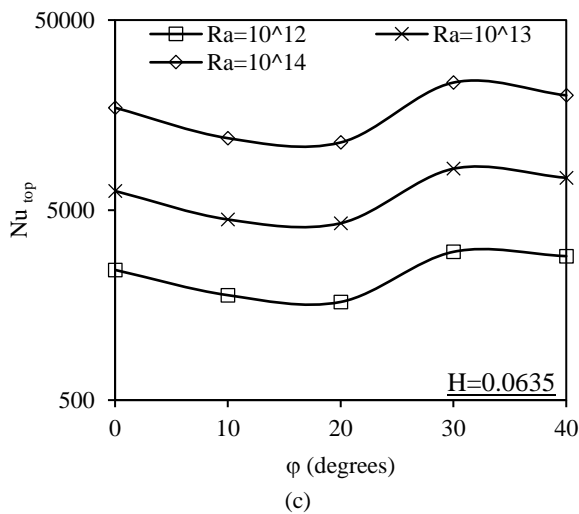
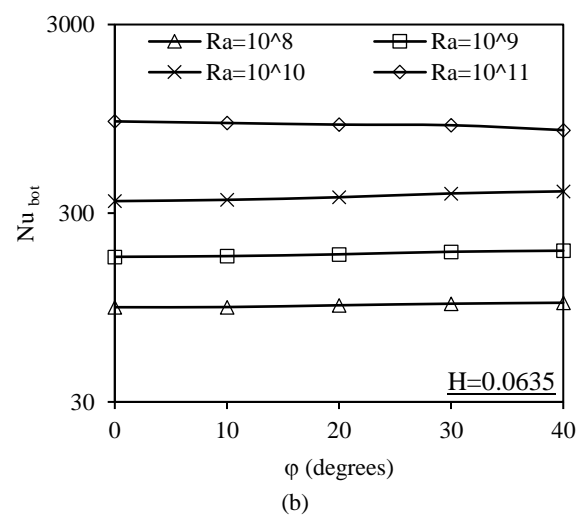
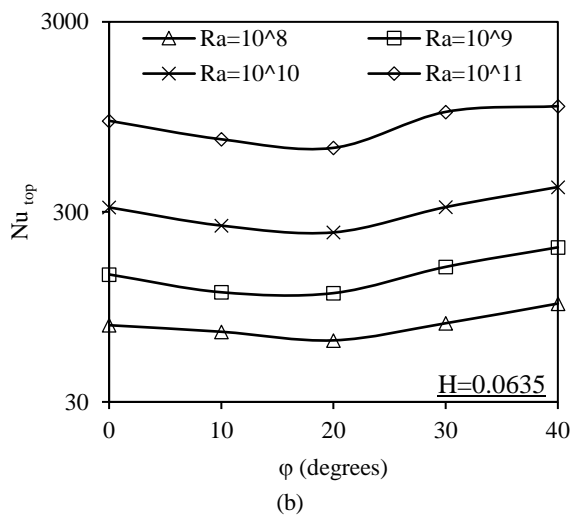
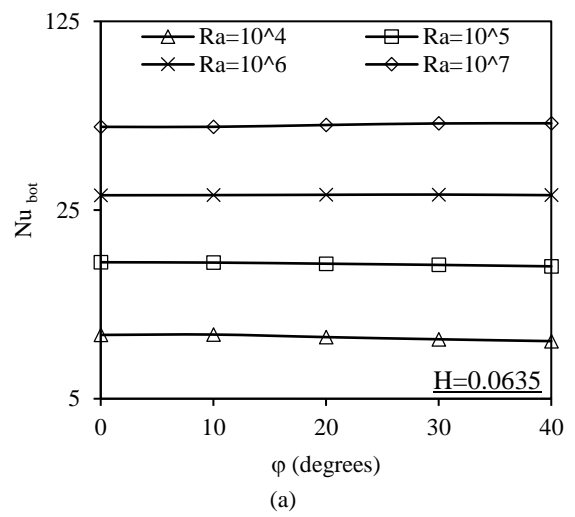
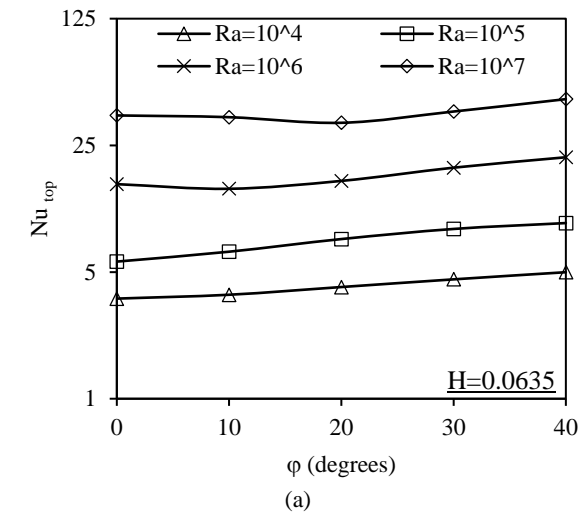


Fig. 6 Variations of the mean Nusselt number for the top surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, H , of 0.0635.

Fig. 7 Variations of the mean Nusselt number for the bottom surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, H , of 0.0635.

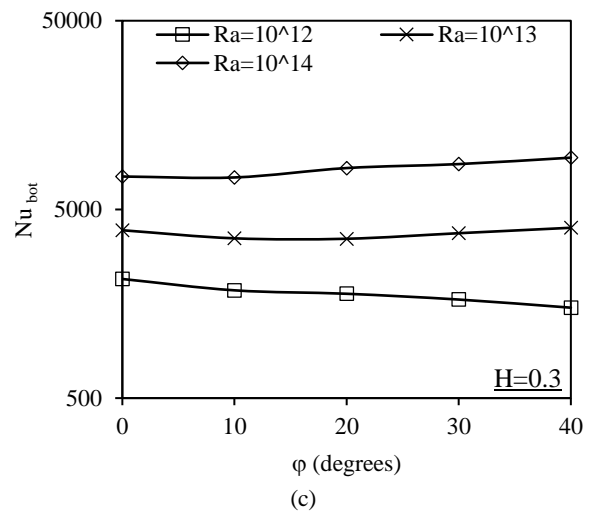
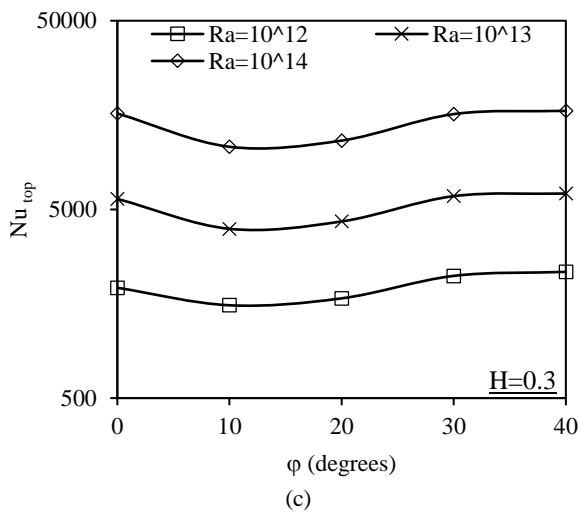
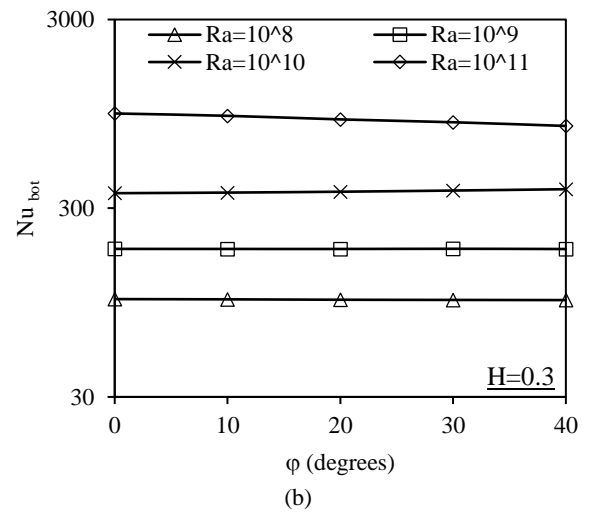
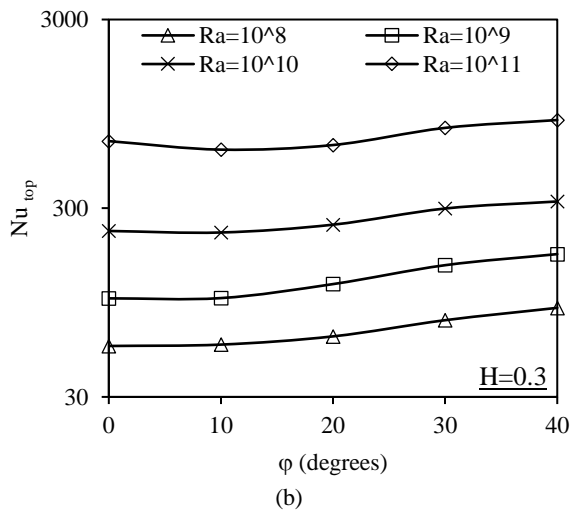
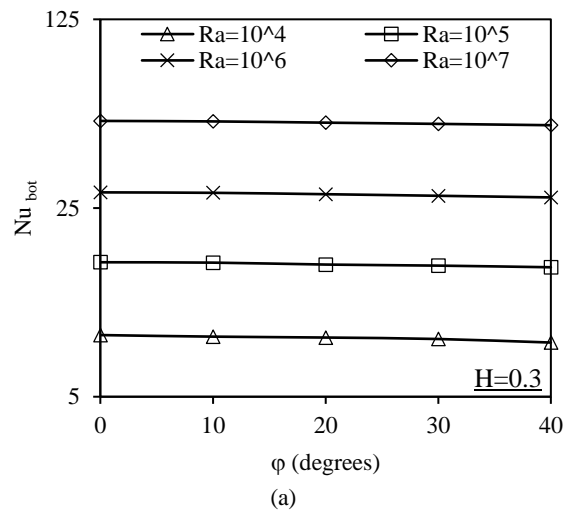
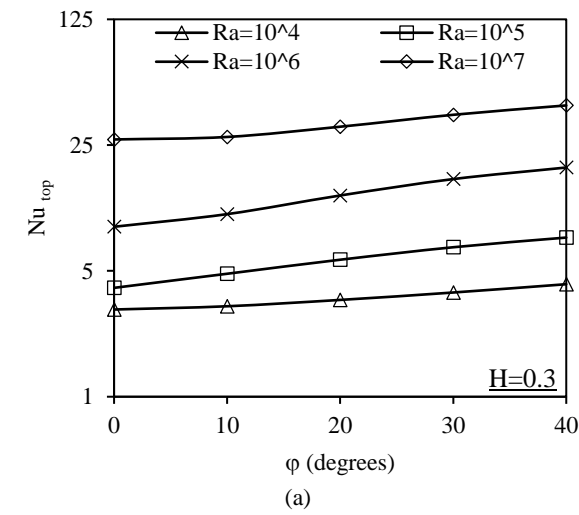


Fig. 8 Variations of the mean Nusselt number for the top surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, H , of 0.3.

Fig. 9 Variations of the mean Nusselt number for the bottom surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, H , of 0.3.

The influence of the diagonal inclination angle on the heat transfer rate from the side surface of the plate is shown by the typical variations of the mean Nusselt number with the diagonal inclination angle given in Figs. 10 and 11 for the dimensionless thickness values of 0.0635 and 0.3, respectively. Again, the Rayleigh numbers range was divided into three subranges. It can be seen from Fig. 10 (i.e., at $H=0.0635$) that the influence of diagonal inclination angle starts at Rayleigh number value of about 10^6 and more specifically when the angle is approximately above 10° where the Nusselt number increases as the diagonal inclination angle increases, this influence being true until the Rayleigh number value of 10^{10} after which the influence of the diagonal inclination angle on the Nusselt number becomes significant after about 20° where the Nusselt number starts to increase with the angle up to about 30° after which the Nusselt number starts to decrease with the angle. For the dimensionless plate thickness, H , of 0.15, it can be seen from Fig. 11 that the influence of the diagonal inclination angle on the Nusselt number starts to be significant at Rayleigh number value of about 10^6 and after about 20° , this influence being true up to Rayleigh number value of 10^8 . For Rayleigh number values of 10^9 and 10^{10} , this influence of the diagonal inclination angle on the Nusselt number is almost negligible. The influence of the diagonal inclination angle on the Nusselt number becomes significant again at Rayleigh number value of 10^{11} . It can be seen from Fig. 11-b, c that the Nusselt number decreases with the diagonal inclination angle for Rayleigh number values of 10^{11} and 10^{12} , whereas for Rayleigh number values of 10^{13} and 10^{14} it will be noted that the Nusselt number first increases as the angle increases from 0° then decreases when the angle is approximately above 30° .

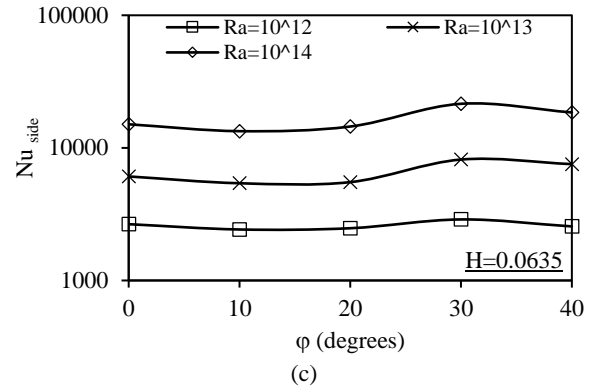


Fig. 10 Variations of the mean Nusselt number for the side surfaces of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, H , of 0.0635.

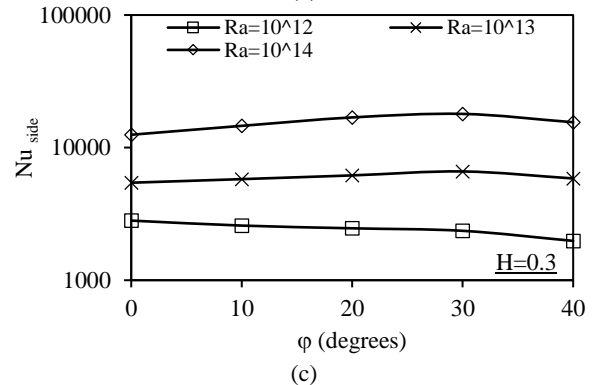
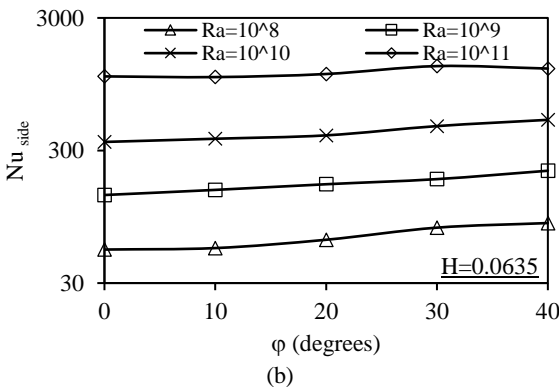
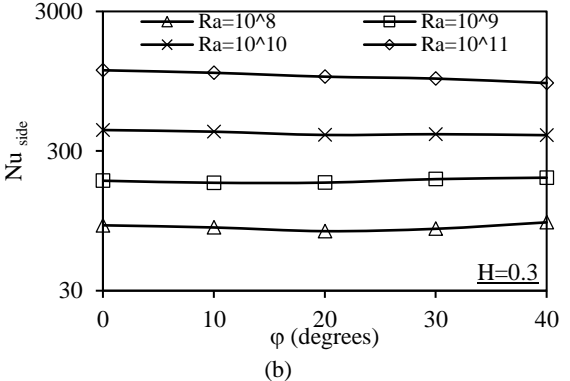
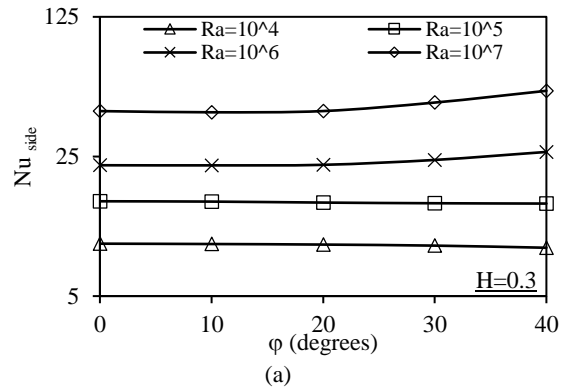
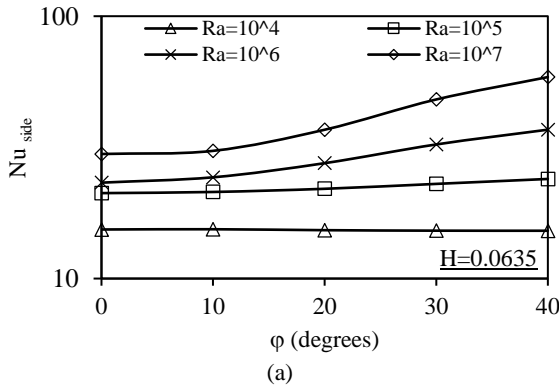


Fig. 11 Variations of the mean Nusselt number for the side surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, H , of 0.3.

4. EXPERIMENTAL APPARATUS AND PROCEDURE

The purpose of the experiments conducted in the present study was to validate the numerical model used to obtain the heat transfer results for the diagonally inclined two-sided square plate. In the experimental study, the heat transfer rate from the plate considered was determined using the lumped capacity method.

4.1 Apparatus

To measure the heat transfer rate from the two-sided plate a support frame on which the plate is mounted was used. The support frame plates were made from a Plexiglas and the supporting elements on which the plate was mounted were made from steel and had a sharply pointed tip to make the contact area with the heated plate as small as possible. The frame assembly was located on a leveled horizontal table. The experiments were undertaken in a large test chamber located inside a closed laboratory to avoid the temperature changes and air disturbance that could interfere with the flow over the plate. The heated plate used in the experimental study was made from aluminum alloy (Al 6061-T6) and had a width, w , of 0.2 m and a dimensionless thickness, H , of 0.0635. Thermocouples, which were previously calibrated were embedded into holes drilled into the side surface of the plate were used to measure the temperature of the plate.

4.2 Experimental Procedure

The heat transfer rate from the square plate in the situation being considered was determined using the lumped capacity method. The method involves heating the plate up using an electric oven to a temperature of roughly 100°C above ambient. The plate was then removed from the oven and allowed to cool to a temperature of about 15°C above ambient. The transient variation of the plate temperature as it cools was measured and recorded over the selected temperature range using a data acquisition system. The cooling time interval was divided into 5 minutes subintervals. From the transient temperature variation, and the specific heat of the plate material, and if the plate temperature was assumed to be uniform at any time instant during cooling process, the overall heat transfer coefficient, h_t , can then be calculated using the following equation:

$$h_t = \frac{mc}{A_{total}t} \ln \left[\frac{(T_i - T_f)}{(T_e - T_f)} \right] \quad (10)$$

To check the plate temperature uniformity, it is usual to assume that if Biot number, Bi , is less than 0.1, the uniform body temperature assumption can be used. In all experimental tests, the values of Biot number obtained were much less than 0.1, therefore, the lumped capacity method could be used with an acceptable accuracy.

The overall heat transfer coefficient given by Eq. (10) involves convection and radiation from plate to the surrounding and conduction to the supporting elements on which the plate is mounted. The conduction heat transfer is assumed to be negligible due to the very small area of contact the plate makes with the supporting frame elements. To correct for radiation, if the body is assumed too small compared with the surroundings to which it is radiating, then the radiant heat transfer coefficient for each time interval is given by:

$$h_r = \sigma \varepsilon (T_m^2 + T_s^2)(T_m + T_s) \quad (11)$$

where, ε is the emissivity of the aluminum plate which is assumed to be 0.1 according to the plate surface finish, T_m is the mean temperature of the plate during the time interval considered, i.e., $T_m = (T_i + T_e)/2$, and T_s is the temperature of the surroundings to which the body is radiating ($T_s = T_f$ for the type of situations being considered here). Equation (11) can, therefore, be rewritten as:

$$h_r = \sigma \varepsilon \left(\left(\frac{(T_i + T_e)}{2} \right)^2 + T_f^2 \right) \left(\frac{(T_i + T_e)}{2} + T_f \right) \quad (12)$$

On average, the radiative heat transfer coefficient was approximately 11 per cent of the total heat transfer coefficient for the diagonal inclination angles considered.

As a result, the convective heat transfer coefficient can be calculated using the following equation:

$$h_c = h_t - h_r \quad (13)$$

The above procedure was applied for each diagonal inclination angle considered, i.e., for $\phi=0^\circ, 10^\circ, 20^\circ, 30^\circ$, and 40° .

4.3 Uncertainty Analysis

The possible errors in various quantities being measured contribute in the uncertainties in the experimental results (uncertainty propagation). An uncertainty analysis was, therefore, performed to give the interval within which an error in the measured value lies. The overall uncertainty in Nusselt number has been determined using the root-sum-square (RSS) method, see (Kline and McClintock, 1953). The relative uncertainty range for the experimental values of Nusselt number at every diagonal inclination angle and over the time interval considered was obtained.

5. EXPERIMENTAL DATA AND COMPARISON WITH NUMERICAL RESULTS

As explained earlier, the experimental data were obtained to validate the numerical model used. A comparison between the numerical results and the experimental data when the dimensionless plate thickness, H , is 0.0635 for various diagonal inclination angles considered is shown in Fig. 12. It should be noted here that the Rayleigh numbers range is a bit different in this figure because of the difference in the ambient temperature during the experiment for each diagonal inclination angle. It can be seen from this figure that for the diagonal inclination angles considered the numerical results obtained lie within the experimental uncertainty (error band of the experimental results) which is at most less than 11 per cent and good agreement was obtained except when the diagonal inclination angle is 40° , the numerical results, however, lie outside of the experimental uncertainty range. This is possibly due to inaccurate prediction by the numerical model of when transition to turbulence starts to occur. Nevertheless, the maximum percentage difference between the experimental data and numerical results for this angle (40°) is less than 14 per cent for the range of Rayleigh number considered. Table 2 shows the mean and maximum percentage differences between numerical and experimental mean Nusselt numbers for various diagonal inclination angles considered.

Table 2 Percentage difference between numerical and experimental mean Nusselt number.

Angle (degrees)	Mean % difference	Maximum % difference
0	1.5964	6.5011
10	1.9647	3.3329
20	1.9970	4.3564
30	1.3126	4.0514
40	10.3058	13.1192

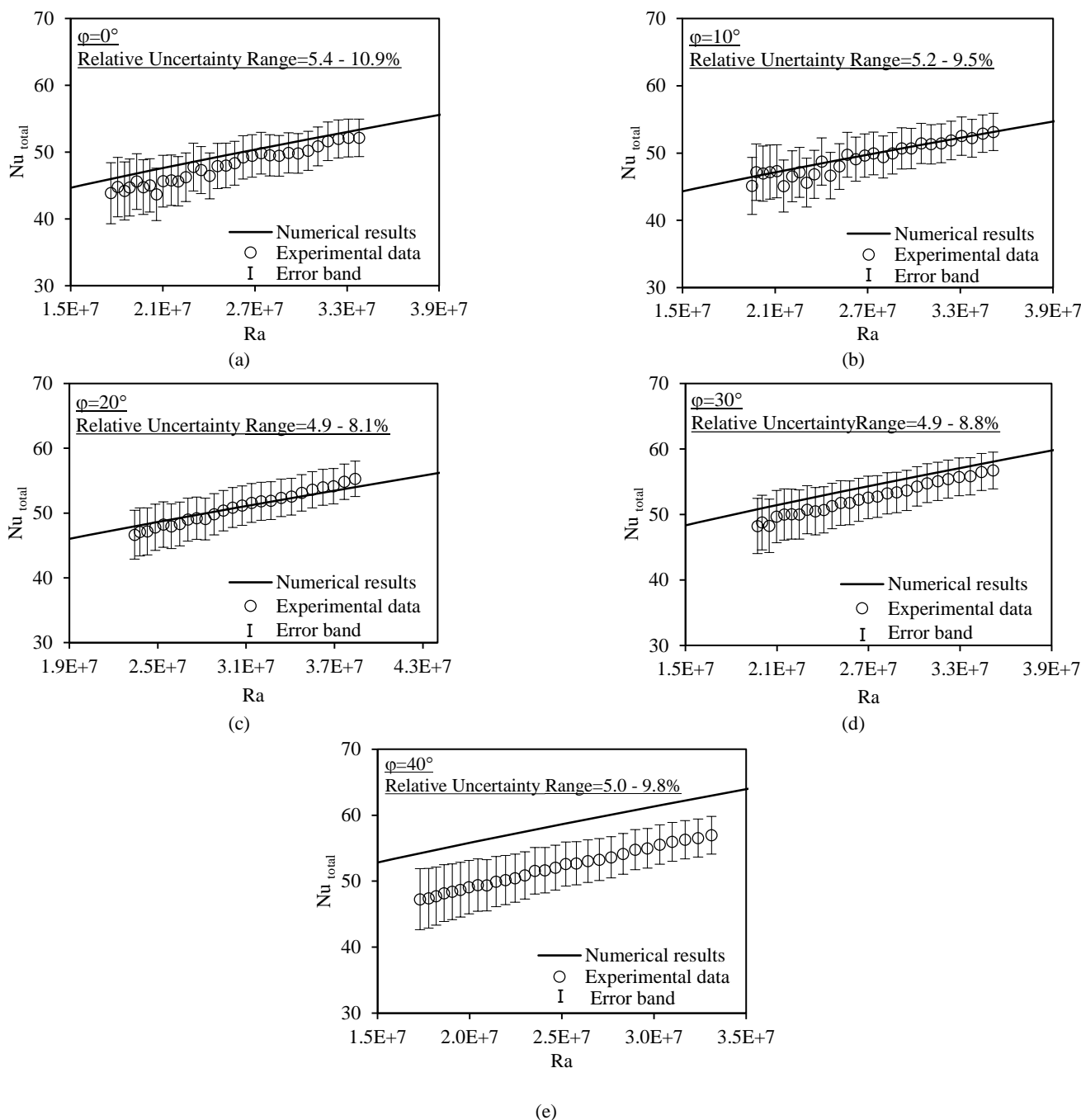


Fig. 12 Comparison of the numerical results with the experimental data for diagonal inclination angle of (a) 0° , (b) 10° , (c) 20° , (d) 30° , and (e) 40° for dimensionless plate thickness, H , of 0.0635. The relative uncertainty range is shown for each diagonal inclination angle.

6. SUMMARY AND CONCLUSIONS

Numerical and experimental results have been obtained for natural convective heat transfer from two-sided diagonally inclined square plates having various thicknesses for various diagonal inclination angles. In all cases it is found that the heat transfer rate from the top surface is lower than that for the bottom surface at the lower Rayleigh number values considered but is higher than that for the bottom surface at the higher Rayleigh number values. The dimensionless plate thickness has an almost negligible effect on the heat transfer rate from the bottom surface but has a significant effect for the top surface especially at lower values of dimensionless plate thickness, the nature of this effect being dependent on the Rayleigh number value

considered. The effect of the dimensionless plate thickness on the heat transfer rate is also significant for the side surface of the plate for the range of Rayleigh numbers considered except at higher values of Rayleigh number. The diagonal inclination angle has a significant effect on the heat transfer rate from the top and side surfaces for the range of Rayleigh numbers considered while this effect is significant for the bottom surface only at higher values of Rayleigh numbers. The numerical results lie within the uncertainty of the experimental data which is at most less than 11 per cent. The only exception, however, is at diagonal inclination angle of 40° where the numerical results lie outside of the experimental uncertainty.

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NOMENCLATURE

A_{plate}	area of the top surface of plate and of bottom surface of plate (m^2)
A_{side}	area of the side surface of plate (m^2)
A_{total}	sum of the areas of the heated surfaces of plate (m^2)
Bi	Biot number
c	specific heat of material from which plate is made (J/kgK)
g	gravitational acceleration (m/s^2)
h	thickness of plate (m)
h_t	total heat transfer coefficient (W/m^2K)
h_r	radiant heat transfer coefficient (W/m^2K)
h_c	convective heat transfer coefficient (W/m^2K)
H	dimensionless thickness of plate, h/w
k	thermal conductivity (W/mK)
m	mass of plate (kg)
Nu	mean Nusselt number
Nu_{total}	mean Nusselt number based on the mean heat transfer rate per unit area over heated surfaces of plate
Nu_{top}	mean Nusselt number based on the mean heat transfer rate per unit area over upper surface of plate
Nu_{bot}	mean Nusselt number based on the mean heat transfer rate per unit area over lower surface of plate
Nu_{side}	mean Nusselt number based on the mean heat transfer rate per unit area over side surface of plate
Pr	Prandtl number
\overline{Q}	total heat transfer rate from the heated surfaces of plate (W)
\overline{Q}_{top}	total heat transfer rate from the upper surface of plate (W)
\overline{Q}_{bot}	total heat transfer rate from the lower surface of plate (W)
\overline{Q}_{side}	total heat transfer rate from the side surface of plate (W)
Ra	Rayleigh number
t	time (s)
T_w	temperature of heated surfaces of plate (K)
T_f	temperature of fluid far from plate (K)
T_i	initial temperature of plate (K)
T_e	final temperature of plate (K)
T_m	mean temperature of plate (K)
T_s	temperature of the surroundings to which body is radiating (K)
w	side length of square plate (m)
Greek Symbols	
ϕ	diagonal inclination angle (degrees)
α	thermal diffusivity (m^2/s)
β	bulk coefficient of expansion ($1/K$)
ν	kinematic viscosity (m^2/s)
σ	Stefan-Boltzmann constant ($W/m^2 \cdot K^4$)
ϵ	emissivity of plate

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