8.5 Film Condensation in Porous Media

Figure 8.19 Film condensation in a porous medium.

Figure 8.29 Film condensation in a porous medium.
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- The dominant forces in the condensation process are gravitational and capillary forces, and the latter dictates the thickness of the two-phase region, $\delta_{lv}$.
- The ratio of gravity and capillary forces is measured by Bond number:

$$Bo = \frac{g(\rho_\ell - \rho_v)K}{\sigma}$$

where $K$ and $\varepsilon$ are, respectively, permeability and porosity.

- When $Bo \approx 1$, condensation in a porous medium is dominated by both gravitational and capillary force.
- The condensation is dominated by capillary force when $Bo < 1$.
  When gravity dominates, Bond number will be greater than 1 and there will be no two-phase region, in which case the analysis will be significantly simplified.
- In the following two subsections, an analysis of gravity-dominated condensation will be discussed first, followed by a discussion of the effect of surface tension on the condensation process.
8.5.2 Gravity-Dominated Film Condensation on an Inclined Wall

- It is assumed that the condensation is gravity-dominated and therefore the liquid and vapor are separated by a sharp interface, not a two-phase region.

- In addition, the following assumptions are made:
  - The condensate film is very thin compared to the length of the inclined wall ($\delta \ll L$) so that boundary layer assumption is valid.
  - The properties for the porous medium, liquid and vapor are independent from temperature.
  - The inclination angle, $\phi$, is small enough for the gravity component in the normal direction of the surface to be negligible.
  - Darcy’s law is valid for both liquid and vapor phases.
Figure 8.30 Gravity dominated film-condensation on an inclined wall in a porous medium.
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Under these assumptions, the governing equations for the liquid layer are

\[
\frac{\partial u_\ell}{\partial x} + \frac{\partial v_\ell}{\partial y} = 0 \tag{8.355}
\]

\[
u_\ell = -\frac{K}{\mu_\ell}(\rho_\ell - \rho_v)g \cos \phi \tag{8.356}
\]

\[
u_\ell \frac{\partial T_\ell}{\partial x} + v_\ell \frac{\partial T_\ell}{\partial y} = \alpha_\ell \frac{\partial^2 T_\ell}{\partial y^2} \tag{8.357}
\]

The boundary conditions are

\[
v_\ell = 0, \quad y = 0 \tag{8.358}
\]

\[
T_\ell = T_w, \quad y = 0 \tag{8.359}
\]
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- At the interface, the boundary conditions are
  \[ T_\ell = T_{sat}, \quad y = \delta_\ell \]  \hspace{1cm} (8.360)

  \[ m'' = \rho_\ell \left( u_\ell \frac{d\delta_\ell}{dx} - v_\ell \right), \quad y = \delta_\ell \]  \hspace{1cm} (8.361)

  \[ m'' h_{\ell v} = - k_{m\ell} \frac{\partial T}{\partial y}, \quad y = \delta_\ell \]  \hspace{1cm} (8.362)

- Combination of eqs. (8.361) and (8.362)
  \[ \rho_\ell h_{\ell v} \left( u_\ell \frac{d\delta_\ell}{dx} - v_\ell \right) = - k_{m\ell} \frac{\partial T}{\partial y}, \quad y = \delta_\ell \]  \hspace{1cm} (8.363)

- Introducing stream function
  \[ u_\ell = \frac{\partial \psi}{\partial y}, \quad v_\ell = - \frac{\partial \psi}{\partial x} \]  \hspace{1cm} (8.364)
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- and the following similarity variables

\[ \eta = \sqrt{Ra_{\ell x}} \frac{y}{x} \]  

(8.365)

\[ \psi = \alpha_{\ell} \sqrt{Ra_{\ell x}} f(\eta) \]  

(8.366)

\[ \theta(\eta) = \frac{T_{\ell} - T_{sat}}{T_{w} - T_{sat}} \]  

(8.367)

- where

\[ Ra_{\ell x} = \frac{(\rho_{\ell} - \rho_{v})g \cos \phi Kx}{\mu_{\ell} \alpha_{\ell}} \]  

(8.368)
The governing equations and the corresponding boundary conditions become

\[ f' = 1 \]  
\[ 2\theta^{''} + f\theta' = 0 \]  
\[ f(0) = 0 \]  
\[ \theta(0) = 1 \]  
\[ \theta(\eta_\delta) = 0 \]  
\[ Ja(\ell) \theta'(\eta_\delta) = -\frac{1}{2} f(\eta_\delta) \]
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\[ \eta_\delta = \sqrt{Ra_{lx}} \frac{\delta}{x} \]  
\hspace{1cm} (8.375)

- is the dimensionless liquid film thickness and

\[ Ja_\ell = \frac{c_p l (T_{sat} - T_w)}{h_{lv}} \]  
\hspace{1cm} (8.376)

- is Jakob number that measures the degree of subcooling at the wall.

Integrating eq. (8.369) and considering eq (8.371)

\[ f = \eta \]  
\hspace{1cm} (8.377)

which can be substituted into eqs. (8.370) and (8.374)

\[ 2\theta'' + \eta \theta' = 0 \]  
\hspace{1cm} (8.378)

\[ Ja_\ell \theta' (\eta_\delta) = - \frac{1}{2} \eta_\delta \]  
\hspace{1cm} (8.379)

- The solution of eq. (8.378) with eqs. (8.372) and (8.373) as boundary condition is

\[ \theta (\eta) = 1 - \frac{\text{erf}(\eta / 2)}{\text{erf}(\eta_\delta / 2)} \]  
\hspace{1cm} (8.380)
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The dimensionless film thickness can be obtained by substituting eq. (8.380) into eq. (8.379)

\[ \text{Ja}_\ell = \sqrt{\frac{\pi \eta \delta}{2}} \exp \left( \frac{\eta^2}{4} \right) \text{erf} \left( \frac{\eta \delta}{2} \right) \]  

(8.381)

The heat flux at the wall is

\[ q''_w = -k_{m\ell} \left( \frac{\partial T_\ell}{\partial y} \right)_{y=0} = \frac{k_{m\ell} (T_{\text{sat}} - T_w) \sqrt{Ra_{\ell x}}}{x} \theta'_\ell (0) \]

(8.382)

and the local Nusselt number is

\[ Nu_x = \frac{q''_w x}{k_{m\ell} \sqrt{\pi} \text{erf}(\eta \delta / 2)} \]

(8.383)

Cheng recommended that eq. (8.383) be approximated

\[ Nu_x = \left( \frac{1}{2Ja_\ell} + \frac{1}{\pi} \right)^{1/2} Ra_{\ell x}^{1/2} \]

(8.384)

Average Nusselt number is obtained by integrating eq. (8.384)

\[ \overline{Nu}_L = \left( \frac{1}{Ja_\ell} + \frac{2}{\pi} \right)^{1/2} Ra_{\ell L}^{1/2} \]

(8.385)
8.5.3 Effect of Surface Tension on Condensation in Porous Media

- The analysis in the preceding subsection is valid for gravity-dominated condensation in porous media ($B_0 \gg 1$).
- When the condensation is gravity-capillary forces dominated ($B_0 \sim 1$) or capillary force dominated ($B_0 \ll 1$), there will be a two-phase region that is saturated by a mixture of liquid and vapor, as shown in Fig. 8.29.
- The fraction of liquid in the pore space is defined as saturation –

$$\gamma_\ell = \frac{\phi_\ell}{\varepsilon}$$  \hspace{1cm} (8.386)

- where $\phi_\ell$ is volume fraction of the liquid in the porous media.
The continuity equation for the two-phase region

\[
\rho \left( \frac{\partial u_\ell}{\partial x} + \frac{\partial v_\ell}{\partial y_2} \right) + \rho_v \left( \frac{\partial u_v}{\partial x} + \frac{\partial v_v}{\partial y_2} \right) = 0
\]  
(8.387)

Mass fluxes for liquid and vapor are governed by Darcy's law

\[
\dot{m}_\ell'' = - \frac{KK_{r\ell}}{\nu_\ell} \nabla p_\ell
\]  
(8.388)

\[
\dot{m}_v'' = - \frac{KK_{rv}}{\nu_v} \nabla p_v
\]  
(8.389)

Vapor flow is negligible compared to the liquid flow so that eq. (8.387) is reduced to

\[
\frac{\partial u_\ell}{\partial x} + \frac{\partial v_\ell}{\partial y_2} = 0
\]  
(8.390)

The velocity components in the x- and y-directions

\[
u_\ell = K_{r\ell} u_D = - \frac{KK_{r\ell} (\rho_\ell - \rho_v) g}{\mu_\ell}
\]  
(8.391)
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\[ v = -\frac{KK_r}{\mu} \frac{\partial p}{\partial y_2} = \frac{KK_r}{\mu} \frac{\partial p_c}{\partial y_2} \quad (8.392) \]

- where \( u_D \) is Darcy velocity. The capillary pressure is

\[ p_c = \frac{\sqrt{K/\varepsilon}}{f(s)} \quad (8.393) \]

- where \( f(s) \) is a Leverett’s function

\[ f(s) = 1.417(1-s) - 2.120(1-s)^2 + 1.263(1-s)^3 \quad (8.394) \]

- and \( s \) is dimensionless saturation defined as

\[ s = \frac{f_s - f_{si}}{1 - f_{si}} \quad (8.395) \]

- The relative permeability in eqs. (8.391) and (8.392) is

\[ K_{r_l} = s^3 \quad (8.396) \]

- Substituting eqs. (8.391-8.396) into eq. (8.390)

\[ 3 \frac{\partial s}{\partial x} + \left[ \frac{\sigma}{\sqrt{K/\varepsilon}} \left( \rho_l - \rho_v \right) g \right] \left[ (3f' + sf'') \left( \frac{\partial s}{\partial x} \right)^2 + sf' \frac{\partial^2 s}{\partial y_2^2} \right] = 0 \quad (8.397) \]
Subjected to the following boundary conditions

\begin{align}
    s &= 0, \quad x = 0 \quad (8.398) \\
    s &= 1, \quad y_2 = 0 \quad (8.399) \\
    s &= 0, \quad y_2 \to \infty \quad (8.400)
\end{align}

Introducing the following similarity variable

\[ \eta = y_2 \left[ \frac{(\rho_\ell - \rho_v)g}{(\sigma / \sqrt{K/\varepsilon})x} \right] \quad (8.401) \]

Eqs. (8.397)-(8.400) are transformed to

\[ s'' = \frac{3\eta s' - 2(3f' + sf'')s'^2}{2sf'} \quad (8.402) \]

\[ s = 1, \quad \eta = 0 \quad (8.403) \]

\[ s = 0, \quad \eta \to \infty \quad (8.404) \]
The governing equations for the liquid film in dimensionless form

\[
\frac{\partial u^+}{\partial x^+} + \frac{\partial v^+}{\partial y^+} = 0 \quad (8.405)
\]

\[
\frac{\partial^2 u^+}{\partial y^{+2}} + 1 - u^+ = 0 \quad (8.406)
\]

\[
\frac{\partial^2 \theta^+}{\partial y^{+2}} = 0 \quad (8.407)
\]

where the dimensionless variables are defined as

\[
u^+_l = \frac{u^+_l}{u_D}, \quad v^+_l = \frac{v^+_l}{u_D}, \quad x^+ = \frac{x}{\sqrt{K}}, \quad y^+ = \frac{y}{\sqrt{K}}, \quad \delta^+_l = \frac{\delta^+_l}{\sqrt{K}}, \quad \theta = \frac{T - T_w}{T_{sat} - T_w} \quad (8.408)
\]

The boundary conditions at the wall are

\[
u^+_l = v^+_l = \theta = 0, \quad y^+ = 0 \quad (8.409)
\]
The boundary conditions at the interface between the liquid film and the two-phase region require that the velocity and shear stress in these two regions match, which makes the solution of the condensation problem very challenging.

Majumdar and Tien (1990) proposed three models to handle the boundary condition at the interface between the liquid and the two-phase region and two of them are discussed below.

**Model 1.** At the interface between the liquid and the two-phase region, the shear stress is zero, i.e., \( \partial \tilde{u}_l / \partial \tilde{y} = 0 \) at \( \tilde{y} = \delta^+_{\ell} \), which is the same as in classical Nusselt analysis.

The velocity profile in the liquid layer is

\[
 u^+_l = 1 - \cosh y^+ + \tanh \delta^+_{\ell} \sinh y^+ 
\]

(8.410)
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The liquid layer thickness can be obtained from an energy balance at the interface

\[
(1 - \text{sech}^2 \delta^+ \ell) \frac{d\delta^+ \ell}{d\tilde{x}} + (1 - \text{sech}\delta^+ \ell) \frac{0.373}{\sqrt{\text{Bo}\tilde{x}}} = \frac{\text{Ja}_\ell}{\delta^+ \ell \text{Ra}_K}
\]  

(8.411)

where

\[
\text{Ra}_K = \frac{K^{3/2}(\rho_\ell - \rho_v)g}{\mu_\ell \alpha_e}
\]

(8.412)

Analytical solution of eq. (8.411) is not possible and it must be solved numerically.

**Model 2.** This model also employs eq. (8.406) to obtain the velocity in the liquid layer except that the boundary condition at \(y^+ = \delta^+ \ell\) is changed to \(u^+_\ell = 1\).

Although it is not as rigorous as Model 1, it is an improvement over Cheng (1981) because it uses non-slip condition at the wall. The velocity profile in the liquid layer is

\[
u^+_\ell = 1 - \cosh y^+ + \coth \delta^+ \ell \sinh y^+ \]

(8.413)
the overall energy balance at the interface
\[
\left(1 - \frac{1}{1 + \cosh \delta^+_\ell}\right) \frac{d \delta^+_\ell}{d \tilde{x}} + \frac{0.373}{\sqrt{Bo \tilde{x}}} = \frac{Ja_\ell}{\delta^+_\ell Ra_K} \tag{8.414}
\]

the local Nusselt number
\[
Nu_x = \frac{\tilde{x}}{\delta^+_\ell} \tag{8.415}
\]

The parameter $R$ in the figure is defined as
\[
R = \frac{Ra_K}{Bo} = \frac{\sigma \sqrt{K \epsilon}}{\mu \ell \alpha_e} \tag{8.416}
\]
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Figure 8.31 Comparison of results of Model 1 and 2 with experiments: (a) aluminum foam metal, (b) polyurethane foam